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ABSTRACT

This is the Teacher's Commentary for Mathematics for the Elementary School, Book K, Special Edition. The writers have relied on the existing SMSG kindergarten and first grade materials as a framework. This special edition is designed to meet the needs of disadvantaged children. Included in the Commentary are background information for the teacher, discussion of activities in the text, and answers to activities and exercises. (RH)

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SCHOOL
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MATHEMATICS FOR THE ELEMENTARY SCHOOL

BOOK K

Teacher's Commentary

SPECIAL EDITION (Revised)

U.S. DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

SMSG

SE 023 010

MATHEMATICS FOR THE ELEMENTARY SCHOOL

BOOK K

Teacher's Commentary

SPECIAL EDITION (Revised)

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PREFACE

The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected and well taught--at all levels, from the kindergarten through the graduate school.

With this in mind, mathematical organizations in the United States cooperated in the formation of the School Mathematics Study Group (SMSG). The general objective of SMSG is the improvement of the teaching of mathematics in grades K - 12 in the schools of this country. The National Science Foundation has provided substantial funds for the support of this endeavor.

One of the prerequisites for the improvement of the teaching of mathematics in our schools is an improved curriculum--one which takes account of the increasing use of mathematics in science and technology and in other areas of knowledge, and at the same time one which reflects recent advances in mathematics itself. Among the projects undertaken by SMSG was that of enlisting a group of outstanding mathematicians, educators, and mathematics teachers to prepare a series of sample textbooks which would illustrate such an improved curriculum.

The development of mathematical ideas among young children must be grounded in appropriate experience with things from the physical world and the immediate environment. The materials in this publication provide for young children an introduction to the study of mathematics that reflects clearly this point of view, in which growth is from the concrete to the abstract, from the specific to the general. Major emphasis is given to the exploration and progressive refinement of ideas associated with both number and space.

The writers have relied on the existing SMSG kindergarten and first grade materials as a framework. However, the writers hope that this special edition will better meet the needs of disadvantaged children.

It is not intended that this book be regarded as the only definitive way of introducing good mathematics to culturally deprived children at this level. Instead, it should be thought of as a sample of the kind of improved curriculum that is needed and as a source of suggestions for the authors of commercial textbooks. It is sincerely hoped that this and other texts prepared by SMSG will lead the way toward inspiring a more meaningful teaching of Mathematics, the Queen and Servant of the Sciences.

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OVERVIEW

A MATHEMATICS PROGRAM BEGINS IN THE KINDERGARTEN.

An informal, intuitive foundation for the development of mathematical ideas begins at "ground level", in the kindergarten wherever one exists within the school organization, or in the first grade if a kindergarten does not exist.

There is good reason to believe that children will be interested in, and will profit from appropriate foundation experiences which emphasize activities in which physical objects from the child's immediate environment are observed, described, and manipulated. The development of each mathematical concept in this book encourages extensive manipulation of sets of physical objects by the children. Your guidance in these experiences will make a significant contribution to the growth of mathematical ideas among the children in your class.

NEW WORDS IN THE KINDERGARTEN.

Most kindergarten children are rapidly and eagerly adding new words to their vocabulary. However, some children will arrive at school having had little or no experience in verbalization. It will take time and a great deal of help from you to get them to feel comfortable with words. As you carefully plan activities in which words are introduced in meaningful situations, yet as naturally and normally as possible, these children will gradually use them more and more.

Just as you are alert to situations which aid in the development of general vocabulary, so you can be alert to contrive and use situations which aid in the development of more specialized vocabulary associated with mathematical learning. For instance, instead of always saying something such as, "Maria, will you please come and sit with the boys and girls at this table," remember often to say, "Maria, will you please join the boys and girls at this table." Or, you frequently may say, "Peter, pick up the blocks that remain on the floor," instead of "Peter, pick up the blocks that are left on the floor." There are many occasions, too, when you will wish to emphasize the meaning of a word in a way such as this: "Joan, do the sets match? Are they equivalent?"

Mathematical concepts are of prime importance. Special vocabulary carefully used to express these ideas becomes a part of the children's vocabulary as they hear you use it frequently and correctly, and as you encourage them to make it a part of their speech. However, the ideas are much more important than the unique vocabulary, and if some children are unable to use this vocabulary, let them express the ideas in words that can have some meaning for them.

NO BOOKS FOR PUPILS.

"Where are the pupil pages?" "Why don't you include examples of seat work for duplicating?" "Isn't this material too easy for the kindergarten?"

These may be questions which occur to you. No workbooks or duplicating pages have been included in this material. Mathematical ideas develop slowly. These ideas can be developed and understood most easily through activities that call for thoughtful manipulation of concrete objects and through activities that make children aware of mathematics in their everyday lives.

With meaningful manipulation of concrete objects, children become active participants and not merely passive watchers or listeners. Their attention is aroused by the use of materials near and dear to them. The game approach catches the interest and enthusiasm of all the children regardless of their intellectual level. It seems to be of special aid to the more immature child and a challenge to the more able child in discovering other relationships new to him. Playing is the serious work of the child this age!

The experiences provided by workbooks and/or worksheets too often make little contribution to effective learning experiences. Too many children become preoccupied with the mechanics of finding and marking the right picture or drawing when the hand and eye coordination necessary for this type of activity is not yet adequately developed. Furthermore, the concepts the children need to understand are difficult to convey in pictures or drawings because they do not permit the actual movement of the objects shown. Action is a key word for children. The active manipulation of objects makes a more lasting impression and contributes to a deeper understanding of the ideas under study.

GETTING STARTED IN THE PROGRAM

Children, their activities, interests, and the possessions which they bring to school provide an ideal starting point for a gradual development of the mathematical concepts and mathematical vocabulary in the kindergarten program. At the beginning of the year, the incidental daily conversations and activities that are closely connected with mathematics help children to become aware of mathematical ideas and vocabulary. However, very early you may find that a definite time can be used when the focus is on mathematics. It is then that the specific suggestions made in this book are intended for your use. Since you are the one who really knows your children and their levels of maturity, you must decide for yourself when to begin using this material. These suggestions should be considered only as a starting point. You may have many excellent ideas of your own that will have meaning and interest for your children in their particular situation.

When you feel that your children are secure and comfortable in the school situation, when their attention span has lengthened, and when they are ready for "new worlds to conquer," then you may want to set aside a regular period of the day when the focus of attention is on some particular mathematical concept. You will need an over-all view of all the material that is to be presented so that you may take advantage of opportunities to use it during the day-by-day kindergarten program.

Your schedule should not be rigid. You may find that in a given week you might devote two or three days to activities suggested in the book and focus entirely on mathematical concepts; another day might be used for incidental discussions or activities using the language but not necessarily pinpointing the mathematical concepts; or there may possibly be a day when no particular attention is paid to mathematics. However, once attention has been focused on a given mathematical concept, take advantage of every possible opportunity to reinforce it.

The illustrative activities need not always follow each other in the printed sequence. You may find that a particular activity may be more meaningful to your children when beginning a new concept than the one listed first. You may not need every activity with all of your children, and you may need additional ones for others. To denote a new activity or a variation in the activity used to develop a particular concept, a • has been placed before the activity. Use your own judgment in selecting the activities that will best focus the attention of your children on the mathematical concept under consideration. However, when working with each concept it will be necessary to progress in an orderly fashion so that new ideas can be

built upon concepts already understood by the children. For example, the child must understand set and member of the set before the idea of subset and removal of a subset can be introduced.

Later in the year it will be possible to use some of the suggestions given in Chapter 8, Additional Activities To Develop Certain Mathematical Ideas in connection with other activities. Then these suggestions need not be the center of focus but can be used to add interest and variety to the activity while at the same time strengthening the geometrical concept. For example, the first activity in the section used geometric figures to designate the seating arrangement when preparing for an activity other than mathematics.

As you read the material in the Commentary, you will realize that you can find or make opportunities daily to introduce, easily and normally, new ideas and new vocabulary. Those which have been introduced already are reinforced by using them in different situations.

On "Sharing Day" children come in with sets: sets of little cars, books, dolls, or other objects. It can be very easy to comment on "the set" that Bill brought and to ask questions about the various "members" of the set without particularly dwelling on the vocabulary.

Daily there are many opportunities to use mathematics during the work period. "Is there a jar (can) of finger paint for each hole at the easel?" "Are there as many hammers as children who want to use them?" "In the set of dishes, is there a plate for each place at the table?" "Is the edge of each plate round like a circle?" etc.

Checking on a one-to-one correspondence is a very real situation during lunch time every day, as it always is when anything is distributed to the class. Is there a glass of milk for every child? Is there a child for each glass of milk?

You might invite another group to join your class to see a puppet show. Your children might join the other classes as they go to enjoy a program in the auditorium.

DEVELOPING CHILDREN'S IMAGINATION

Helping a child to develop his imagination is a daily, on-going activity in the kindergarten. You use it to add interest, fun, and zest to his daily life. Developing the child's imagination helps him enlarge his vocabulary so that he has understanding of the words he hears and takes this understanding to his formal reading in the first grade. This imagination also plays an

important part in the child's ability to understand mathematical concepts and ideas. For example, since you want him to think of the number three as that number common to all sets having three members, he must be able to imagine many, many sets containing three members each, and many different types of sets (three pieces of candy; three boys; a boy, a dog, and a fishing pole; a father, a mother, and a child; etc.). He must be able to imagine these in many circumstances. As he works later with a set of points, he will need imagination to perceive these sets of points, and he will need it to be able to perceive a line going "on and on."

Many of your children will be unable to imagine anything and will completely lack interest in the activity. For these children activities depending upon imagination should come in small doses and infrequently at first. The book suggests activities in which pupils will pretend to be characters from a story they liked or in which pupils are asked to see pictures in their minds to strengthen a certain concept. Using the imagination can be made to be an enjoyable experience for most children.

Enlarging and developing this imagination can be a source of great pleasure to both the children and their teacher and can do much to add warmth and real group feeling to the class situation. We all need to remain alert to take advantage of situations (or to create these situations) that will provide this growth in imagination. We must keep in mind, however, that it is difficult (if not impossible) for young children to imagine things that have not first been experienced directly or in a recognizably related context.

OBJECTS CHILDREN MIGHT BRING FROM HOME FOR "SHARING TIME"

A young child is eager to explore and to learn more about his environment. The things he brings to school to "share" with the others often point out some of his special interests.

Objects that are brought in by the various children may range from a favorite toy or trinket to rusty nails and old tin cans. Something of interest can be and must be found in each thing a child brings. The rusty nail can lead to an interesting discussion on the likenesses and differences between that nail and a new nail of similar size brought from the workbench in the room. The old tin can can be discussed as to size, shape, and use, and might lead to a discussion and an experiment to see whether or not it will hold as much, more than, or less than another container that is being used in the room.

Since these "sharing time" objects are often of much interest to all the children, they can sometimes be used to provide the center of focus on mathematics for the day.

"Janet brought her new doll, and Margaret brought her Raggedy Ann. Is one doll taller than the other doll? How can we find out?"

"Larry has brought a set of horses and Lynn a set of cowboys." "Is there a horse for each cowboy? Is there a cowboy for each horse?" "What can we do to find the answer?" "Yes, let's have Larry and Lynn pair the members of their sets so that we can all see and know the answer."

Sometimes your planned activity for the day will need to be postponed in order to make immediate use of these items that have some special interest to the children. Whenever they relate to the concepts you are trying to develop, use them and wait to use your prepared material another day. In working with the mathematics program, as in every other phase of the kindergarten work, you must be alert to take appropriate advantage of every learning situation as it arises.

SETS.

A set is simply a collection of things. The things belonging to a set are called its members. A set may be defined by some property common to its members (the set of all books on this shelf; the set of all children in this room). Sets may also consist of quite unrelated objects and may be defined simply by listing their members, as in the set whose members are a certain boy, a certain book, and the moon.

It is possible for a set to have only one member. For example, the set whose only member is the teacher of the class, or the clock on the wall, or the classroom piano. There is also one special set called the empty set which by definition has no members at all. For example, the set of all live elephants in the classroom is the empty set.

One set is a subset of another if every member of the first set is also a member of the second. There is no member of the first set that is not a member of the second set. Thus, the set of girls in a classroom is a subset of the set of children in that classroom; and the set of all tricycles is a subset of the set of all toys. There are two special cases of the definition of subset. First, any given set is a subset of itself (because it fits the definition of subset: every member of a given set is also a member of the set!). Second, the empty set is a subset of every set (because the empty set has no member that is not a member of a given set).

COMPARISON OF SETS; NUMBERS.

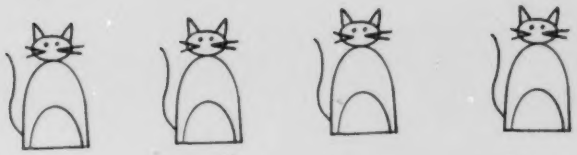
We can compare two sets by pairing the members of one set with those of the other. Consider the following example.

In this example, there are members of the set of stars that are left unpaired with members of the set of circles. So, the set of stars has more members than the set of circles. Alternately, the set of circles has fewer members than the set of stars.

When the pairing process is such that there are no members left in either of the two sets, we say that the two sets have been placed in one-to-one correspondence. The two sets are equivalent.

When two sets are equivalent, they have the same number of elements. The number of elements in a set is the property shared by all sets equivalent to it. Thus, four is the property of all sets equivalent to:

or to:



It is easy to set up a one-to-one correspondence between these sets, showing that the set of birds is equivalent to the set of cats (and vice versa). The number of birds (4) is equal to the number of cats (4). However, the sets are not equal. Two sets are equal only if they have exactly the same members, and a set of four cats is a very different thing from a set of four birds.

When a set A has more members than another set B, we say that the number of members in set A is greater than the number of members in set B and that the number of members in set B is less than the number of members in set A. Thus the ideas of more than and fewer than for sets lead to the ideas of greater than and less than for numbers. The relation of "fewer than" or "more than" permits us to rank or order several sets of different numbers of members. The set with fewest members is selected first, then the other sets are arranged so that each succeeding set has more members than the one before it. The set having the most members comes last in order.

JOINING AND REMOVING SETS.

To join a set A to a set B means to form a new set whose members are all of the members of set A together with all of the members of set B. This new set is called the union of the two given sets.

Example:

Set A: {a, b, c}

Set B: {p, q}

Union : {a, b, c, p, q}

In this example, the first set and the second set happen to be disjoint. This means that the two sets have no members in common. When this happens, the number of members in the first set plus the number of members in the second set is the number of members in their union: so, we obtain the addition fact:

$$3 + 2 = 5.$$

Thus, the joining of disjoint sets serves as an approach to the adding of numbers.

In the same way, removing from a given set one of its subsets serves as an approach to subtraction. For example:

Given set: {a, b, c, p, q}

Subset removed: {p, q}

Remaining set: {a, b, c}

The number of members in the given set minus the number of members in the subset removed is the number of members in the remaining set.

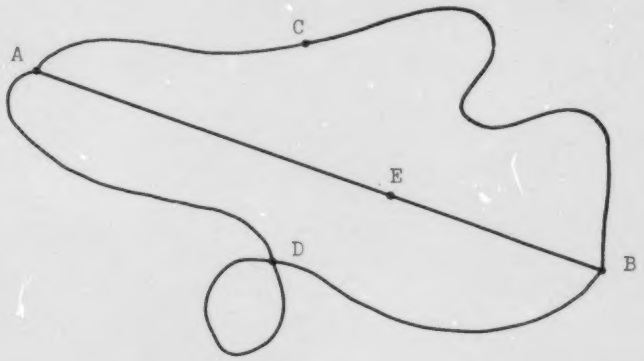
From this illustration, we obtain the corresponding subtraction fact

$$5 - 2 = 3.$$

You are reminded that sets are joined while numbers are added; a subset of a set may be removed while numbers are subtracted. Joining and removing are set operations; addition and subtraction are number operations.

GEOMETRIC FIGURES.

The following diagram represents several geometric paths.



One of these paths passes through point C and has points A and B as its endpoints. The path that passes through D intersects itself at that point. The path that passes through E is the most direct path having A and B as endpoints. This particular path is called a line segment.

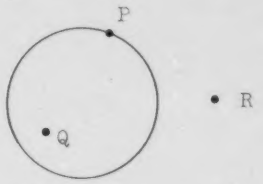
We may think of tracing a closed path that starts at C, that passes through B, that intersects itself at D, that passes through A, and that returns to C. We also may think of tracing a simple closed path by starting at E, then passing through A and C and B, and returning to E -- without intersecting itself (except at E).

Many familiar geometric figures such as circles, triangles, rectangles and squares are particular kinds of simple closed paths, as illustrated below.

A simple, yet satisfactory model of each may be constructed from wire, a pipe cleaner, starched string, or similar material.

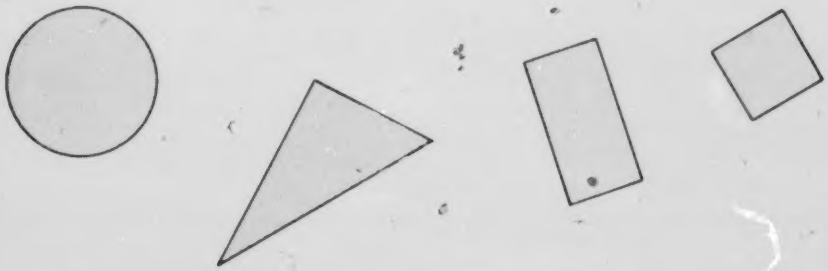
For each kind of figure, there are many points on that figure or path, there are many points inside that figure or path, and there are many points outside that figure or path--all of which lie within the same flat surface.

In the representation at the right, P is a point on or of the circle; Q is a point inside the circle; and R is a point outside the circle. The circle itself does not include either Q or R.



The "roundness" of a circle distinguishes it from a triangle or a rectangle or a square. A triangle is distinguished by its three sides (segments) and the three points where pairs of sides meet. A rectangle is distinguished by its four sides, by the four points where pairs of sides meet, and by the unique way in which pairs of sides meet.* A square is a rectangle. It is a special kind of rectangle in which all four sides are exactly the same "length." A copy of each side would fit exactly on every other side of the figure.

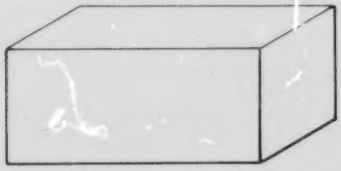
Sometimes we are interested in a simple closed path and all the points within that path. The union of a simple closed path and its interior is called a region, as illustrated below.



* In a rectangle, pairs of sides that meet at a point suggest that which we call a "right angle." However, the concepts of "angle" and "right angle" are more sophisticated than we can develop and use with kindergarten children.

We distinguish among regions in terms of the paths which are their boundaries. The boundary of Figure A is a circle, and Figure A represents a circular region. The boundary of Figure B is a triangle, and Figure B represents a triangular region. Figures C and D are each representations of rectangular regions, with Figure D representing the special instance of a square region.

Young children's earliest experiences with models of geometric figures involve things such as balls; "round" cans and plastic containers; boxes, and the like; or with toys and blocks that have such shapes. Some such simple closed surfaces have segments as their edges and plane regions as their faces or bases. For instance, each edge of the box represented below is a segment, and each face is a rectangular region.



With kindergarten children, we approach the study of geometric ideas through the use of models of simple closed surfaces, and from these we derive concepts of circle, triangle, rectangle, square, and their associated regions.

"Bruce, you brought an interesting set of animals today. As you talk about it, please hold up the members of your set one at a time so that we may see each animal as you tell us about it." "You have already made many new friends in the set of children in our room. Bill, can you name all the members in our set?" After set and member have been used informally for a period of time, children may be led naturally to the idea that a set may have only one member or even no members at all.

Familiar examples of one member sets may be the set of pianos in the classroom, the set of clocks on the wall, the set of teachers in the class, or the set of American flags in the room. As for sets with no members, reference may be made to such sets as the set of elephants seated at the teacher's desk. Also, you may refer to such sets as the set of crayons in a crayon container when there are no crayons in the box.

Deliberately, we have used the word "set" almost to the exclusion of other words, such as group, bunch, collection, etc. This was done to illustrate the wide variety of situations to which the term can be applied. However, do not conclude that the intention was to suggest that words such as groups, bunches, etc., not be used, they should be used when it is natural to do so.

You are encouraged to bring in the use of terms naturally. Although the guide may suggest, "Will the members of the set of boys in our class stand?", don't feel that you should always say that. Sometimes it is best to say, "Will the boys stand?" In brief, you can try so hard to use these terms in so many activities that they may become too artificial.

As the school year progresses, set descriptions should become more explicit. Instead of referring to the set of blocks in general, you may want to refer to the set of small blocks on the bottom shelf. The need for more exact descriptions may be better understood from the following examples: When a child is asked to bring you the set of red pencils on your desk, a specific set of pencils is identified. When asked to bring simply a set of red pencils, he may return with any set of red pencils which he collects in the room.

ACTIVITIES TO HELP DEVELOP THESE CONCEPTS:
SET.

The playhouse is most likely a familiar sight in your classroom. Materials from it may be used for early discussions of sets of objects. In particular, the set of dishes may be most meaningful. Place the playhouse table and chairs in front of the children. Have a set of dishes, a set of

Chapter 1

SETS

OBJECTIVE: To develop the concept of set and the use of set terminology, including the idea that a set may have only one member or even no members at all.

VOCABULARY: Set, collection, member, set with one member, single, set with no members (empty set).

MATERIALS: A variety of materials, such as: books, toy cars, toy trucks, blocks, pencils, crayons, sheets of paper, paint brushes, scissors, game boxes, puzzles, beads, paper clips, pegs, balls, playhouse furnishings, etc.

A variety of materials to be used on magnetic or flannel boards, such as: stars, trees, fruits, animals, storybook characters, models of circular, triangular, and rectangular regions.

Magazine pictures: family, automobiles, telephones, clothing, food, toys, planes, trains, boats, and any other appropriate illustrations of collections of objects. Storybooks about a family, and others for making set references.

BACKGROUND NOTES:

Set concepts can be effectively developed when the vocabulary is used incidentally (but reasonably naturally!) throughout the school day. Informal situations occur where reference can be made to particular sets of objects and their members. For example, "Billy, please put your set of blocks away. Children, will those of you who have blocks that belong to Billy's set put them back?" Michael "lines up" with the set of boys because he belongs there or is a member of that group.

As sets are introduced, the terms, member and members of a set, may come naturally into the conversation. For example, "Here is a new set of zoo animals for the room. Jeanne, can you name all the members of this set?"

silverware, and a set of glasses or cups available. Identify each of these collections as a set. These questions can be used as a guide:

Who has helped to get the table ready for dinner?
What did you put on the table? (plates, glasses, silverware, etc.)

Let's pretend that we're going to have dinner and need to get our table ready. Jim, will you put the plates on our table?

Point to one of the plates. Ask if this plate belongs to the set of dishes on the table. (Yes.)

John, what would you like to put on the table for us? (Possibly the silverware.)

Ask if a particular knife or fork or spoon belongs to the set of silverware.

Julie, what set do we still need to put on the table? (Glasses.)
Will you please put a set of glasses on the table?

Now is our table ready for dinner? (Yes.)

Let's have some people come to eat the dinner. Can we call this a set too? (Yes.)

Name four children to be those who go to eat at the table. Ask if the other children belong to the set named to eat.

Finish the discussion by identification of other sets in the playhouse, at home, etc.

MEMBERS OF A SET.

Select a story about a family and read this to the class. Some suggestions are listed below:

Flack, Marjorie	The Story of Ping
McCloskey, Robert	A Day in Maine
	Make Way for Ducklings
	Blueberries for Sal
Ward, Lynd	The Biggest Bear
Zion, Gene	The Plant Sitter
(traditional)	Goldilocks and the Three Bears
	Little Red Riding Hood
	Cinderella

After reading the story, have the children identify various sets described in the story. When the sets have been identified, the attention can be focused

upon the things that are members of each set. The sets may range from the set of buildings on a particular street to the set of people or animals about whom the story is written. Draw the discussion to a particular group of people or animals in the story. Make references to this group with questions or statements similar to the following.

Today we read a story about a family.

The family was a group of people (animals, etc.).

Is that a set? (Yes.)

Who belonged to this set?

Who were the members of this set? (Members of the set should be listed.)

Cut-outs of characters in the story may be used. Then have children touch one member of the set, then another, etc., to help reinforce the concept of members of a set. Dramatization of the story may help them understand members. Statements selected from the ones which follow may be adapted to your particular use.

Karla, name all of the members of the set of people (animals, etc.) in our story.

Yes, all of these are members of the set of people (animals, etc.) in our story.

We say that each person or thing that belongs in the set is a MEMBER OF THE SET.

You children are members of our class.

You belong to the set of children in our room.

You are also members of the set of children who come to our school.

If children seem to have difficulty remembering the story, let them look back at the pictures in the story.

•Call attention to the fact that there are different sets of toys in the classroom.

We have many sets of toys in our room.

Let's look at the toys and name the sets we can see. (Wheeled toys, boats, dolls, blocks, puzzles, Lincoln logs, etc.)

Michael, what is your favorite set of toys? (Boats.) The boats are fun, aren't they?

Please put all the members of the set of boats on one of the tables.

Martha, what is your favorite set of toys? (Dolls.)

Please put all the members of the set of dolls on another table.

Did Michael find all the members of his set? (Answer depends on what he did.)

Did Martha find all the members of her set? (Answer depends on what she did.)

Ask similar questions about other sets of things in the classroom [deleting any reference to "favorite" sets in most instances]: sets of picture books, crayons, blocks, beads, paste sticks, paint jars, paint brushes, etc.

- Talk further about the sets previously used, identifying the members of these sets (e.g., the set of dishes in the playhouse; each plate, each cup, and each saucer, etc., is a member of this set). Then ask if the table is a member of the set of dishes.

- Point out that some things are not members of specified sets. For instance, the books are not members of the set of paint brushes. Steve is not a member of the set of girls; the trucks are not members of the set of blocks, etc. Have children point out other objects in the room that are not members of some sets.

SETS WITH A SINGLE MEMBER.

Today, let's have all the girls wearing red dresses stand.

All the boys wearing black shoes stand. (Continue with a variety of set descriptions. The teacher should be seated.)

Now will the members of the set of boys stand? (Be seated.)

Members of the set of girls stand? (Be seated.)

Members of the set of teachers stand? (The class should react at this point.)

Yes, I am the only member of the set of teachers in this room (assuming that there is not more than one teacher).

Can you think of any other sets in our room with just one member? (The set of pianos, the set of teacher's desks, the set of clocks, the set of American flags, etc.)

Sets can have more than one member, such as the set of children in our room.

Sets can also have just one member.

We call a set having just one member A SET WITH A SINGLE MEMBER.

(The word single may be new to most children. Discuss set with one member and set with a single member as having the same meaning to help clarify the understanding of the term single.)

Mention other sets which have a single member, such as: the set of swimming pools or playgrounds at a particular location in your city; the set of principals at your school; the set of automobiles that some families own, etc.

SETS WITH NO MEMBERS: THE EMPTY SET.

On the day of this discussion the teacher might wear a piece of clothing containing at least one pocket. It should be empty. (The children will search this pocket for a set of things.) Before the activity, select as "secret helpers" at least three children who have clothing with pockets. Place some small objects in a pocket of each child's clothing. Tell them that they will be asked to empty their pockets when the game begins. After the class is together, call the "helpers" up, one at a time. Inform the children that the pockets contain surprise sets and ask them to describe the set as it is placed on the table. The following may be helpful:

Bruce, will you please empty your pocket for us?

Let's look at the things Bruce had in his pocket. What are the members of this set? (Paper clips, rubber bands, buttons, beads, etc.)

Repeat the above for each of the "secret helpers." Make it amusing and fun by using the game idea. After each helper has emptied his pocket, select a child to come up and empty the teacher's pocket.

What are the members of the set of things in my pocket? (No members.)

You mean my pocket is empty! There is nothing in it.

This helps us to think about the set with no members.

We call a set having no members THE EMPTY SET.

Ask the children to suggest other examples of the set with no members, but do not be concerned at this time if the idea needs further development.

- The following activity may be used as a variation of the previous one. Whereas the first procedure actually involved the removal of the sets from the pockets, the following is a game of "pretend."

Before the lesson starts, prepare four boxes instead of pockets. Select three children and tell them that each is to pretend that the things in the box given him were things that came out of his pocket last night. The fourth box is empty, representing the teacher's pocket.

We are going to pretend that each of these boxes contains the things that Mary, Michael, Kathy, and I removed from our pockets last night.

Mary, show the class the set of things you had in your pocket. (E.g., doll shoe, penny, bobby pin, and button.)

What are the members of Mary's set? (Class describes.)

Things found in other children's boxes are described in a similar way.

Now, let's see the set of things I had in my pocket last night.

Mark, describe the members of this set. (Child finds teacher's box empty.)

You mean my pocket was empty? I guess I had nothing in my pocket last night.

There were no members in my set so I had THE EMPTY SET.

ADDITIONAL ACTIVITIES

1. Have the children observe sets of objects in the room and describe them using the terms: set, member, etc. (e.g., sets of chairs, tables, crayons, paint brushes, pictures, blocks, etc.)

2. Daily directions are given for moving children from one activity to another, for their participation in some special activity, and for dismissal. From time to time, use the new vocabulary to refer to the group under consideration. For example,

   Will members of the set of girls wearing ribbons in their hair please get their hands washed for lunch?

   Will members of the set of boys wearing brown shoes be the first to get ready to go to the library?

   Will all the girls who are five years old please get ready to go home?

3. The set of days in the week, the set of days we go to school, the set of weeks in the month, the set of months in a given season, the set of months in a year, etc., may be mentioned from time to time as the calendar is used. The set of hands on a clock face may be mentioned in connection with telling time.

4. Initiate a class project of creating a science display with collections of rocks, sea shells, leaves, butterflies, etc.

5. Preparation for the work period: When selecting or assigning work activities, separate the children according to sets (e.g., the set of children who wish to paint, the set of children who wish to draw, etc.). Note: At a later date

this experience may be used for "pairing" children with easels, brushes, paper, etc.

6. Make perception cards using pictures from magazines. Show these to the class and have children describe the sets of objects and name the members of each set.

   Have children cut out pictures and make their own cards.

7. Display wooden stand-up figures in the classroom and have the children classify the objects, e.g., sets of farm animals, wild animals, community helpers, members of the family, etc.

8. Read stories about people, animals, things, etc., and describe the sets and the members of the sets. Also have children name objects which are not members of these sets.

9. Have children go on a scavenger hunt around the room. Place about 20-25 cut-outs around the room and ask the children to find them. Have them describe the sets they find. Let pupils who found none describe their set as the empty set.

10. Prepare "surprise boxes" which contain different objects. Have children sort the objects into sets and describe the sets. These boxes may be used to develop other concepts in the future; e.g., boxes of buttons may contain several different colored buttons. The children may sort these into sets of each color, sets of large buttons, sets of small buttons, etc.

11. Give pupils envelopes or some other suitable container to take home and ask them to bring back a set of objects that they find on the way home or to school. Have them describe their sets to the class.

12. Play a game of musical chairs and describe members of the set of children with chairs, the set left without a chair, etc.

13. There may be a store in the room; if so, use this as another approach to help clarify the idea of sets of objects, members of sets, etc.

14. Some children may wish to paint or draw pictures of sets and organize their own "Set Pictures" or "Set Scrapbook".

15. "What's Missing?" Place several sets of objects on a table -- books, blocks, cards, and scissors. Ask a child to describe each set.

    Lori, can you describe these sets for me?

    Yes, a set of books, a set of blocks, etc.

Now close your eyes while we remove one of the sets. (Remove the set of books.)

Look at the table again, Lori. Can you tell us which set is gone?

This game can be varied by selecting different objects for each game, such as ceramic figures, geometric figures, things from the playhouse or the store. Members of the class may also enjoy preparing the sets and directing the activity.

Chapter 2

RECOGNIZING GEOMETRIC FIGURES

OBJECTIVE: To help children to recognize geometric figures (circle, rectangle, triangle, and square), and regions bounded by such figures.

VOCABULARY: Circle, rectangle, triangle, square, figure, inside, outside, on, side, edge, region, solid.

MATERIALS: Geometric figures (curves and regions) made from wire, string, pipe cleaners, paste sticks, sandpaper, construction paper, felt or flannel, yarn, straws; box tops (circular, rectangular, and square); triangle from rhythm instruments; rings; macaroni; fruit jar rings; plain, round bracelets; rope; flannel board; pegboard, pegs, rubber bands or elastic thread, styrofoam.

BACKGROUND NOTES:

Children learn to recognize the shapes of objects about them, identifying those objects shaped like triangles, rectangles, circles, etc. They may become acquainted with these shapes by moving their fingers around the edge of such objects as a domino, a card, a record, coins, and cut-outs from cardboard and plywood. Through such activities, they begin to distinguish between the objects which have a feeling of roundness and those which have corners. The children should discover that the objects which have corners also have sides; that an object having sides will always have more than two; and that the object will have as many sides as it has corners. Children begin to associate the idea of triangle with a 3-sided object, circle with an object that is round like a coin, and rectangle with an object that has four sides and corners like a sheet of paper.

Activities are designed to help children to learn to distinguish between a region, its boundary, its inside and its outside. Blocks and other solid models can be used to advantage when distinguishing regions (faces) from boundaries (edges).

ACTIVITIES TO HELP DEVELOP CONCEPTS OF CIRCLE, RECTANGLE, TRIANGLE, AND SQUARE:

Have available many familiar objects from the playhouse, store and other places in the room (e.g., blocks, plates, boxes, cans, wheels on toys, records, books, balls, etc.). Provide opportunities for the children to handle the objects and discuss their shapes. These experiences should help the children distinguish the relationships among geometric curves, regions, and solids. Begin the discussions with familiar objects and gradually introduce the idea that the terms circle, rectangle, and triangle refer to the edges of the solids.

When a child looks for one of the curves, say a rectangle, he is most likely to find examples such as the base of a block, a cupboard door, the top of the table, or a pane of glass in the window or door. Each shows not only a rectangle but also its interior. Again, with each region emphasize that it is the edge that is a rectangle.

USE OF REGIONS

Before the class arrives, place around the room several construction paper models of regions. Have all of the children seated for this lesson.

> We are going to play a game. (Hold up a circular figure.)
> I want to see how quickly you can find two figures that have the same shape as the one I'm holding.
> They may be the same size or they may be larger or smaller than this one.
> They may or may not be made of the same colored paper.

Do not mention that you are holding a circular figure. Some children may call this circular region a circle. At this point, do not make an issue of this. However, merely refer to it as a circular shape or a cut-out whose "border" is shaped like a circle.

Repeat the game using triangular and rectangular regions.
Have many types of triangular regions represented. Have the rectangular regions displayed in various positions and be sure to include some rectangular regions that are square.

- The following activity may be used as a variation of the previous one.

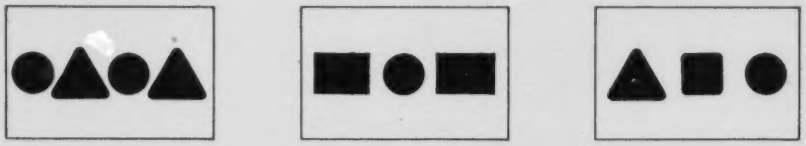
Have several felt models of regions in a box on the table with the flannel board.

> Look at the figure I am placing on the flannel board.
> Karla, see how quickly you can find four figures in the box that are shaped like the one I put on the board.

When figures are located, see if they can be identified by members of the class. Put these figures on the flannel board.

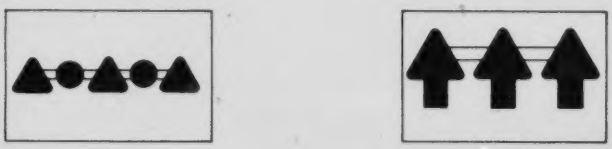
Repeat the game using other regions.

- Give the children seven circular regions of the same size to arrange in designs. They may be delighted to find that if six are arranged in a ring, there will be room to fit the seventh in the center.
- Make available colored disks of various sizes and shapes. Let the children paste these on sheets of Manila paper, newsprint, or brown paper bags from the grocery store to form designs.
- Using geometric regions, start various patterns on the flannel board, and have pupils continue them, e.g.,



Form similar patterns using kindergarten beads, macaroni, straws, blocks, paste sticks, etc.

- Cut various figures from colored construction paper and have children paste them on a larger sheet to create a pattern design. For variety, these can be done on long rectangular pieces of construction paper.

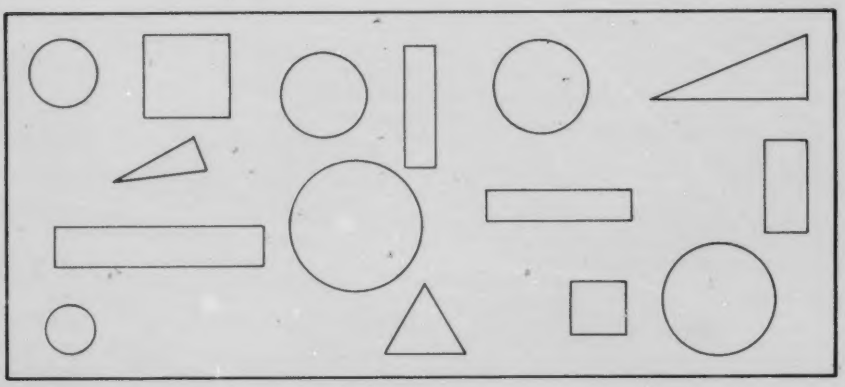


- Some children may enjoy drawing and coloring regions. Then they may cut and paste them to create pattern designs.
- Some children may need other experiences to help them identify the various regions. Have many cardboard models of circular, rectangular (including square) and triangular regions. Keep the models in a shopping bag. Let the child reach in, choose a model and without removing it from the bag, move his fingers around the edge. Have him describe the model and identify it. Then let him find other models like the one he chose. Now have him find models different from the one he chose and use the same method to see if he can identify them. This can be successful as a team activity using one child who can identify all of the regions and another who is having difficulty.

CURVES

On the board, place at random models of circles, triangles, rectangles and squares. Do not crowd the models. Include different sizes and colors. These models are to be cut-outs rather than regions. All of the figures except the circle may be easily made from wire, pipe cleaners, straws and elastic thread or construction paper. For models of circles, you may use fruit jar rings, plain round bracelets, styrofoam rings, embroidery hoops, tires from wheel toys.

You should start with a display that includes only five or six models; then over a period of time include a greater number of models, as illustrated.



Watch what I choose from the board.
When I stop, I will point to someone to come and take off the one he thinks I should take next.

Without another word, take a circle, another circle of a different color, and another of a different size. Select a child to remove the next shape. If he doesn't pick another circle, say,

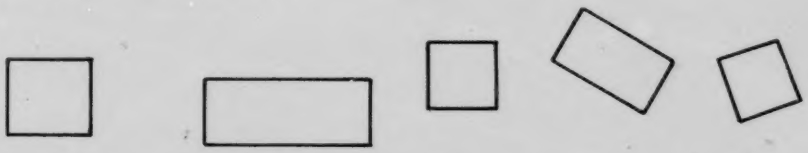
No, that is not what I would have taken next.
Jim, show us what you would have chosen.

Put the cut-outs back. Again remove circles of different size and color. Have another child remove the next shape. At no time call these figures circles. After a few such experiences, explain or let a child explain what is happening to any child who has not yet figured it out for himself. If the figures are called circles, agree but do not force the word at this time. The child doing the explaining may say that "they are all round".

Then let one of the children pick out three models of circles and select a child to remove the model he thinks would be chosen next.

At another time, use the same models and remove the ones that are triangular.

At another time use the same models and remove some members of the set of rectangular models. The set you remove may look like this:



From the set of rectangular models, remove the set of squares. Ask how these models are alike and different from the members of the set of rectangular models. (Intuitively, the children should learn that squares are a special kind of rectangle.)

- Have on display models of circles. Ask if anyone knows the name of the figure. Supply the name if necessary. Let the children run their fingers around the edges of the models. Ask the children to find objects in the room that are shaped like a circle. Ask them to tell about objects outside the classroom that are shaped like a circle.

Similar activities can be planned using models for triangles and rectangles including special rectangles called squares.

Models for triangles and rectangles can be made from the materials previously mentioned. However, the pegboard with pegs and colored rubber bands or elastic thread is very satisfactory. You can display a model of a rectangle or triangle and have children make models like the one you made. The triangles from the rhythm instruments may be used as models.

Provide opportunities for the children to manipulate squares and rectangles that are not squares to help them distinguish between the two shapes.

Have children select all the squares from a set of rectangular models. Point out that the square is a special kind of rectangle. See if they can tell why this is true.

There may be many objects in the room that are shaped like circles, triangles, squares, and rectangles. Give the children ample practice identifying objects shaped like all of these. Whenever possible, reinforce the idea that the term rectangle, triangle, etc., refers to the edges of the shape about which we are talking. The figure is the outline.

USE OF SOLIDS

In a bag, place some solid geometric figures (square, rectangular, triangular, and circular). Wooden models are very good; however, other objects may prove equally satisfactory (boxes, cans, etc.). Hold a newspaper between a child's eyes and hands. Ask him to reach in, pick out one object, feel it, and tell what shape it is. This same procedure may be changed by asking the child to feel for and pick out a designated shape.

From a set of rectangular shaped boxes, have a child or children to pick out all the square boxes.

If your floor is tiled, call attention to the square regions that make up the tiled floor.

Geometric figures cut out of sandpaper help reinforce the "feel" of the shape for some children.

Continuous review experiences may help the children gain a better understanding of geometric figures.

USE OF REGIONS AND SOLIDS

Before the children arrive, put different sized circular, rectangular, and triangular shapes (blocks, boxes, cans, and circular, triangular, and rectangular felt regions, tagboard cut-outs, etc.) around the room. When the children are seated, hold up a circular figure. Have the children identify it. Give them about three minutes to see how many circular objects they can find. Hold up another geometric shape and proceed as before. Continue until all the shapes are used. (Be sure that some of the rectangular shapes are square.)

From where they are seated, have the children visually check to see that the right shapes have been found each time.

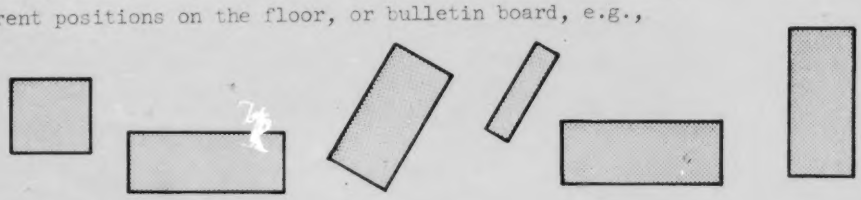
- Put an object into a bag and describe it as to shape and use (e.g., it is circular in shape. It is hard, smooth, and rather flat. Food is put on it.). After a child guesses what is in the bag, he may feel the object to determine whether or not he still thinks he has made a good guess. If he is not right, let another child guess and then feel the object. Some objects might be: a small block, a plate from a dollhouse, a small drum, a paper cup, a cracker, etc. Some children may be able to describe to the other children the object or model that is in the bag. The models could include some of the smaller tagboard and construction paper cut-outs, triangle from the rhythm instruments, etc.

- "I'm Thinking Of ..."

  I'm thinking of something in this room.
  It is shaped like a triangle and is made of wood.
  What is it? (A triangular-shaped block.)

  I'm thinking of something that is shaped like a rectangle.
  It is made of metal.
  What is it? (The tray of an easel.)

- To reinforce the concept that a rectangle is a rectangle regardless of the position in which it is seen, use many models of rectangular regions of various sizes made from tagboard or colored construction paper and place in different positions on the floor, or bulletin board, e.g.,



One by one, ask various children to find a figure shaped like a rectangle and place it in front of him.

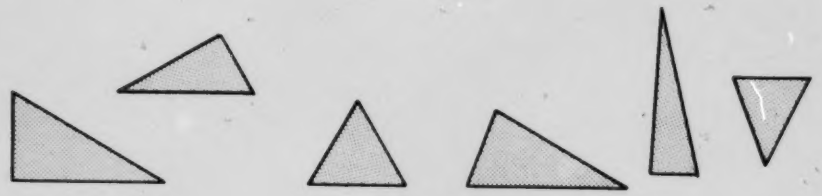
The figures that you chose were put on the floor in just any position.

Did it make any difference how they were placed or were they still all shaped like rectangles?

Pick up your rectangular shape and put it down in a different position.
Is the figure still shaped like a rectangle?

Now pick up your rectangular shape and turn it over.
Is the figure still shaped like a rectangle?

The above procedure might be used for triangles. Here again, it is important that many sizes be used and that they be placed in various positions.



- Display models of each of the figures, circle, rectangle, triangle, and square in various positions. Call on one child to identify all of the figures. If he is successful, he is permitted to rearrange the figures and call on a classmate to continue the game. Children should be allowed to manipulate the figures if necessary and to make the correct identification.
- Reserve a section of the bulletin board or some special part of the room for displaying models and designs that were created as the children studied the geometric figures. They may have made animals, toys, people, mobiles, etc., using various shaped regions.

INSIDE, OUTSIDE, AND ON

Many activities must be provided whereby children learn to distinguish and describe points or locations according to whether they are inside, outside, or on a given circle, triangle, or rectangle.

Make a rope circle on the floor and have the children stand around it. Ask one or two pupils who understand the meanings to help you demonstrate your directions for inside, outside, or on.

Stand INSIDE the circle.
Stand ON the circle.
Stand OUTSIDE the circle.

After the helpers return to the group, tell the class that you are going to give some quick directions and they are not to let you fool them.

Boys stand inside the circle.
Boys stand on the circle.
Boys stand outside the circle.
Girls stand inside the circle.
Girls stand outside the circle.
Girls stand on the circle.
Everyone stand on the circle.
Jim, Jack, Susan, and Mildred stand inside the circle. Etc.
Everyone stand outside the circle.

Later this could be used as a "Simon Says" game.

- Large geometric figures might be formed on the floor by pulling yarn taut and using masking tape to hold the corners in place. Inside, outside, and on games and directions can be used with these easily seen figures.

  Using string or yarn and chalk, make a circle on the floor. Each child will be given a marble as he is seated around the circle. From where he sits, or kneels, he gently rolls his marble toward the circle. As his marble stops, he calls out its position with respect to the circle: "inside", "outside", or "on". After each child has had his turn, have all the marbles "outside" put into one set, those "on" into another set and those "inside" into the third set. (The "on" set may be empty!)

- Later in the year: After a study of comparison and order, this activity can be repeated and the children can decide which set has the most members and which the fewest. These sets can then be ordered with the set with fewest members on the left, the middle set next, and the set with the most members on the right. Pairing can be used to decide the order if there is any doubt.

- Ask the children to stand around a circle made of wire, yarn, clothesline or a hoop and take turns tossing a beanbag into the circle. Have each child tell whether the bag lands inside, outside, or on the circle.

This game could also be used as an out-of-doors activity using a tire, hoop, or rope for the circle.

- The concepts of inside, outside and on can be extended by "Let's Pretend" games using children and objects. With a hula hoop or large model of a circle made of rope, pretend the class is at a rodeo. Place the hoop or rope on the floor or table. Have enough of the small rubber or plastic cowboys, women, children, and men in other occupations in a bag so that each child may reach in and quickly pull out an object. He will then play the part of that figure. The cowboys can go inside the circle with the horses to be ridden, some of the children can sit on the circle fence, and the others can stand where they wish outside the circle while the rodeo is in progress.

- In another "Let's Pretend" game, the children might go to the circus. Small plastic or rubber circus animals could be put in a bag and each child would act the part of the animal he pulled from the bag. Each animal would go inside the circus ring as it was his turn to perform. The children who were not playing the part of an animal would watch from outside the ring.

- Give each child a 9" × 12" sheet of paper on which is drawn a circle, a rectangle, and a triangle. Also give each a small object (small plastic car, animal from farm set, etc.). Ask each to describe his toy to the others.

  Give directions such as:

  Place your set INSIDE the circle.
  Place your set ON the triangle.
  Place the set of cars OUTSIDE the rectangle.
  Place the set of pigs ON the circle.

  If possible, have some of the children give directions for placing the objects.

- Many singing games call for a circle. Call attention to the group forming the circle, the child who is inside the circle, and those who are outside the circle.

  Some of the games are: "Froggie in the Middle", "The Farmer in the Dell", "Bow Belinda", "In and Out the Window", "Looby Loo", "The Old Brass Wagon", and "Hokey, Pokey".

  Circle games can be used in the same manner, e.g., "Duck, Duck, Goose" and "Drop the Handkerchief".

ADDITIONAL ACTIVITIES

Many oppurtunities may be provided whereby children will see geometric figures and objects shaped like geometric figures in different positions and under varying circumstances. A few ideas for activities of this type are suggested here.

1. Have one child draw one of the geometric figures on the chalkboard. Have another child draw a different figure inside the first. Have a third child draw a different figure outside the first figure. This may be a good way to review the figures.

2. Place yarn or elastic thread on a pegboard in the shape of a Christmas tree. Have available regions of various shapes that will be decorations and some solid geometric figures that will be presents. Let the children place the decorations either inside or on the tree and the presents outside the tree.

3. Have several models of the various shapes arranged in a circle on the floor. Ask one child to step inside the circle. Blindfold him and spin him around. When the blindfold is removed, he is asked to identify the figure in front of him. Then he must go to the display board and find a figure having the same shape.

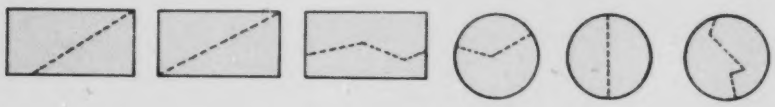
4. Make a box having all edges 4 inches. Draw various shapes on the sides. Let the children roll the box and identify the shape which appears on top. The child is then asked to match the shape with the same shape on display.

5. When the children enter the room, pin a paper region on each one. As the day progresses, have all the children wearing rectangular regions engage in a given activity. Have special activities for the ones wearing circular, triangular, and square regions.

6. Some of the puzzles may provide opportunities for the children to handle regions and feel edges.

7. You may use riddles to have the children locate objects of a particular shape in the room.

8. Have circular and rectangular regions cut into two parts. Place all of the pieces in a box Let each child reach in the box and select a cut-out. He must then find the child who has the piece that will fit with his to make one of the regions. This activity may be used to find a partner when going for a walk, to the playground, to the library, to the auditorium, etc., e.g.,



9. "What is Missing?" Game
Put four figures in a row on the magnetic board. Have the children name the shapes in order from left to right. As they close their eyes, the teacher removes one of the figures. One child must tell which one is missing. This may be continued for a few minutes using different arrangements of the materials.

10. The following books may be used as motivation or as an introduction to the lessons in geometry. They should be reread several times during the year to strengthen the concepts and to encourage the pupils to become more aware of things around them. These books may also stimulate the imagination.

Borten, Helen	Do You See What I See?
Budney, Blossom	A Kiss Is Round
Kohn, Bernice	Everything Has a Shape
Schlein, Miriam	Shapes

Chapter 3

COMPARISON OF SETS

OBJECTIVES: To help children understand that sets may be compared by pairing their members and that the size of the members in the sets is not relevant to the comparison.

To develop some ability to describe the results of pairing by using the terms as many members as, more members than and fewer members than.

VOCABULARY: Pair, match, as many as, equivalent, more than, fewer than.

MATERIALS: A variety of set materials such as: paint brushes, paper, crayons, tools, name cards, objects for magnetic or flannel board, paper clips, buttons, bottle caps, blocks, rhythm instruments, small toys, small animals, pegs from peg board, yarn, spools, bags, dominoes, perception cards.

BACKGROUND NOTES:

The process of pairing is applied to members of sets. It is a process whereby a member of one set is associated with a member of another set. When we ask a child to pair the members of two sets (in so far as it is possible to do so), we want him to associate members of the two sets on a one-to-one basis. The basis for pairing is quite arbitrary, and any member of the one set may be paired (associated) with any member of the other set. Notice that pair is used as an active verb, and not as a noun, as in the expression "a pair of shoes," etc. In fact, you should avoid using things like "a pair of mittens" or whatever when developing the idea of pairing members of sets. A more appropriate idea to use might be that of "partners," if this is a familiar one to children. This is suitable, however, only if partners are formed by associating a member of one set with a member of another set. The purpose of using "pairing" in this restricted sense is to give a basis for comparing two sets by indicating whether one set has exactly as many members as another set, or whether it has more members than another set, or whether it has fewer members than another set.

We also use the word match in a special way that is distinct from other connotations that children may have for this word. We say that two sets match -- or that they are matching sets -- if the members can be paired on a one-to-one basis and there are no members "left over" in either set.

Several of the activities suggested in this section will be facilitated if children are able to distinguish left from right. This is not a mathematical ability, of course; and you will wish to develop it independently so that children can apply it to their mathematical learning. If the ability to distinguish left from right is developed in conjunction with the presentation of mathematical concepts, the child's attention too often is focused on the mechanics of the activity rather than on the mathematical content involved. If some children are not able to distinguish between left and right by the time you use some of the suggested activities, you will need to circumvent this by pointing out exactly where you want things placed on a flannel board, etc.: "Put the members of one set here (pointing). Put the members of the other set there (pointing)." "Put the members of one set on this side (pointing). Put the members of the other set on that side (pointing)."

ACTIVITIES TO HELP DEVELOP THESE CONCEPTS:

AS MANY AS.

Use daily activities to illustrate pairing the members of one set with members of another set. Be sure to have children identify the sets before beginning the activity. For example, pair the pupils with sheets of paper for art activities, or with rhythm instruments, toys, etc., asking:

Is there a sheet of paper for each child?

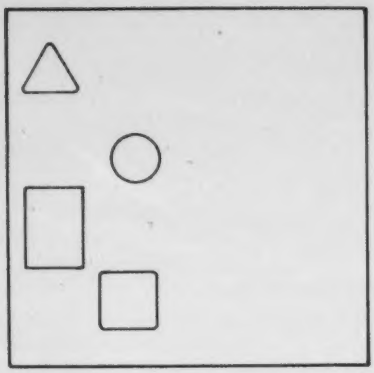
Is there a child for each sheet of paper?

Is there a child for each instrument (or toy)?

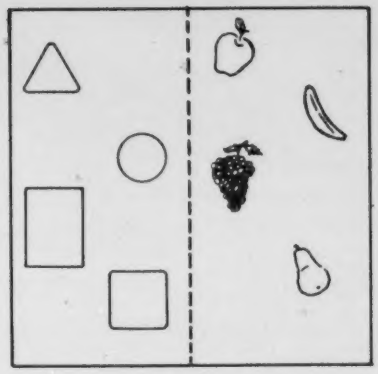
Is there an instrument (or toy) for each child?

Daily snack time and class parties offer a wealth of opportunities for pairing: napkins with straws, favors with refreshments, refreshments with place mats, place mats with children, etc.

- Select a child to place a set of objects on the left side of the magnetic or flannel board. (See Picture 1.) Limit the objects to be used since the members of two sets will have to be paired. Have the class describe the set. On the right side of the board, place another set with as many members as the first set. Have this set described too.



Picture 1



Picture 2

We now have two sets on our board. (Indicate.) (See Picture 2.)

Let's see if there are AS MANY MEMBERS in one set AS there are in the other.

Who can show us how we might do this?

Explore with the children whatever procedures are suggested, giving particular attention to the idea of pairing. If no one suggests the essence of this idea, you will need to do so. Regardless of whether it is suggested by the children or by you, emphasize the nature of the process in a way such as this:

John, you pick (choose, take, etc.) a member of the set of shapes and put it here (pointing to a spot close to the "top center" of the flannel board).

Sue, you pick a member of the set of fruit and put it there (pointing), next to the member that John picked. Good.

You have PAIRED a MEMBER of the set of shapes and a MEMBER of the set of fruit.

Can we pair more members in this way?

Maria, you may pick a member of the set of shapes.

Tom, you may pick a member of the set of fruit.

PAIR the members and put them here (pointing to a spot below the members paired previously).

Continue in this way with the rest of the members. Then, when all members have been paired as in Picture 3, ask questions such as these:

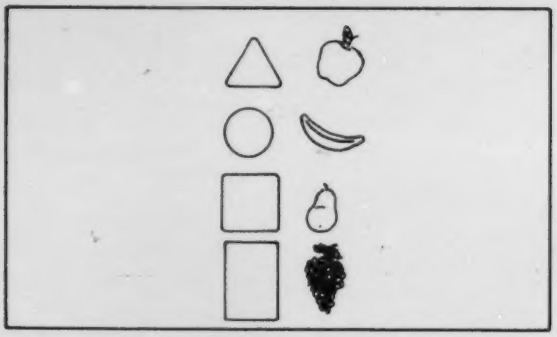
Is each shape paired with a piece of fruit? (Yes.)

Is each piece of fruit paired with a shape? (Yes.)

Now we know that each set has AS MANY MEMBERS AS the other set.

The set of shapes has AS MANY MEMBERS AS the set of fruit.

The set of fruit has AS MANY MEMBERS AS the set of shapes.



Picture 3

Have other equivalent sets ready to use.

Let's ask Bill, Kathy, and John to play the game.

Bill, you can choose a set of things to put on the left side of the board.

Kathy, you choose a set to put on the right.

Now, John, will you pair the members of the two sets?

Repeat this game until each child has had a turn or as long as interest is maintained. Each time, emphasize that there are as many members in one set as there are in the other.

In due course of time you will find it convenient to bring into conversation several terms that apply when one set has as many members as another set: match, matching, equivalent. If one set has exactly as many members as another set, you may say that:

the two sets MATCH; or that
they are MATCHING sets; or that
the two sets are EQUIVALENT; or that
they are EQUIVALENT sets.

Your introduction and use of such terms should not be forced or hurried. Major concern is with the concept that is first expressed by the words as many members as. The vocabulary of match, matching and equivalent should be used only to the extent that these words facilitate the development of that concept. It is important that children understand your use of the terms; but it is not

particularly important or necessary that children use the terms readily in their own conversation. It is sufficient if children understand a question such as, "Do the sets match?" or "Are the sets equivalent?" It is not essential that children themselves be able to say, for instance, "The set of blocks is equivalent to the set of dishes."

•The following activity may be terminated at A, or it may be continued to B, or it may be continued to C. One day you may wish to go only as far as A, and then on later days continue to B and then to C.

Prepare in advance boxes, bags or envelopes containing two types of set materials (buttons, small blocks, felt cut-outs, small toys, etc.) for each child. Be certain that there are as many members in one set as there are in the other. At this stage, the children may be more successful if the sets contain no more than six members.

Begin somewhat in this way:

I have put a set of flags on the left side on the magnetic board and a set of ducks on the right.

Glenn, will you pair these two sets as we did the other day?

Is there a flag for each duck? (Yes.)

Is there a duck for each flag? (Yes.)

Yes, there are as many members in one set as in the other.

Then proceed with the boxes, etc., of prepared materials.

Today, let us PAIR members of some sets that I have put in your boxes (envelopes or bags).

Now, let's look inside the boxes (bags, etc.).

What different kinds of objects did you find? (Have them named.)

Yes, you have a set of (buttons) and a set of (blocks).

Take the blocks out and put them on the right side of your desk.

Now, put the buttons on the left side of your desk.

Now, pair members of your set of blocks with the members of your set of buttons.

Is there a button for each block? (Yes.)

Is there a block for each button? (Yes.)

Are there AS MANY members in one set AS there are in the other? (Yes.)

Are you sure?

Were there any buttons left over? (No.)

Were there any blocks left over? (No.)

A

When the members of one set are paired with members of another set and there are no members left over, we say that the sets MATCH.

Do the sets of buttons and blocks match? (Yes.)

How did we find out if the set of buttons had AS MANY members AS the set of blocks? (We paired the members of the set of buttons with the members of the set of blocks.)

B

We may say that the set of buttons is EQUIVALENT to the set of blocks. This means that the sets match.

Each set has AS MANY members AS the other.

C

- The above activity may be used during free time. Children might enjoy selecting sets of their own. Simple sets can be formed from: shapes of various colors of construction paper, pegs from peg boards, beads, pieces of yarn, small farm animals, small cars, trucks, etc.
- Place five chairs in a row. Select five children and ask them to be seated in the chairs. Point out that the children are paired with the chairs and that there are as many children as there are chairs and there are as many chairs as there are children. The set of children is equivalent to the set of chairs.
- Ask children to show that they have as many fingers on one hand as on the other; stress that they cannot count the fingers but must show you. If needed, suggest that they pair their fingers by putting the finger tips from opposite hands together. Thus, each finger is paired and there are no fingers not paired. The set of fingers on the left hand has as many members as the set of fingers on the right hand.
- Have the children pair the fingers on one of their hands with the fingers on the hand of a classmate.

- Your children are probably familiar with the story of "The Three Bears". Reread or review the story. Then ask questions such as:

  Was there a bowl for each bear?

  Was there a bear for each bowl?

  Was there a chair for each bear?

  Was there a bear for each chair?

  Was there a bed for each bear?

  Was there a bear for each bed?

  What can we say about the set of bears and the set of bowls?

  Yes, there are as many members in the set of bears as there are in the set of bowls.

  The sets match or are equivalent.

  Ask similar questions about the other sets. For example,

  Were there as many people in the story as there were bears?

- As the children are seated, give each a set containing several members. (Have a different number of members in the sets for children sitting side by side.) Along the length of the floor mat, scatter small objects so that each child may find enough members to form a set that is equivalent to his.

  Today, you each have a set. On the mat you see many objects that can be used to make other sets.

  Let's see how quickly you can pick up a set of the objects near you and pair them with the members of your set.

  You must make a new set that has AS MANY MEMBERS AS the set I gave you.

  Be sure that your new set is EQUIVALENT to the set that you already have.

- Instrumental music time may be used to reinforce the concept of pairing. This activity is one in which all members of the class can become involved, provided you have an instrument for each child.

  Is there an instrument for each child? (Yes.)

  Is there a child for each instrument? (Yes.)

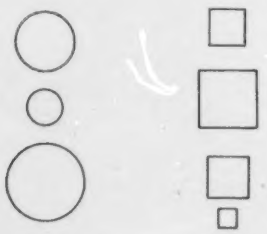
  Do the set of instruments and the set of children MATCH? (Yes.)

  How do you know? (There is an instrument for each child and a child for each instrument. There are no children left over. There are no instruments left over.)

What can we say about the sets?

Yes, they are equivalent. There are as many members in one set as there are in the other set.

- Provide many opportunities for children to identify sets that are not equivalent. Display two sets that do not match. Have children pair members of the two sets, in so far as it is possible to do so, and in this way show that the two sets are not equivalent. Suppose, for instance, that you displayed a set of circle cutouts and a set of square cutouts, such as



After pairing has been carried as far as possible, ask:

Is there a circle cutout for each square cutout? (No.)

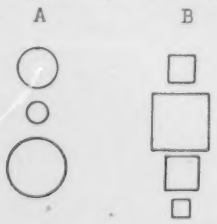
Is there a square cutout for each circle cutout? (Yes.)

Does each set have as many members as the other set? (No.)

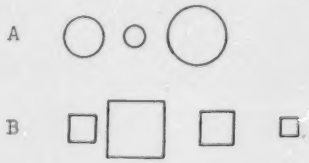
Do the sets match? (No.)

Is the set of circle cutouts equivalent to the set of square cutouts? (No.)

- When displaying sets on a flannel board or magnetic board, sometimes place one set above or below the other, rather than to the right or left of the other. Children need to be able to pair members of sets when they are displayed in a variety of positions. As a transition, after some experience with pairing members of sets that have been arranged vertically, follow immediately by using the same pair of sets, this time displayed horizontally, asking some of the same questions as before. For example, Arrangement 1 followed by Arrangement 2 below:



Arrangement 1



Arrangement 2

MORE THAN.

Pin some identification (a piece of colored yarn or ribbon or a geometric shape, etc.) on ten girls and ten boys when they enter the room. When you are ready for the lesson, ask the boys with the special identification to stand on one side of you and the girls on the other side. Then, pair a member of one set (the set of boys) with a member of the other set (the set of girls).

What have I done with the members of the set of boys and the members of the set of girls? (Paired.)

Suppose Jack and Jane are partners. You may ask:

Jack, name the girl who is your partner.

Jane, name the boy who is your partner.

Ask the other members of the class who are seated,

Does every girl have a boy as a partner? (Yes.)

Does every boy have a girl as a partner? (Yes.)

What can we then say about this set of boys and this set of girls? (There are as many boys as there are girls. The sets match. The sets are equivalent.)

- Repeat the activity but have more members in one set than in the other. Suppose you decide to have more boys. When the pairing has been completed, ask:

Does every girl have a boy as a partner? (Yes.)

Does every boy have a girl as a partner? (No.)

Does each set have as many members as the other set? (No.)

Do the sets match? (No.)

Which set has some members that are NOT paired? (The set of boys.)

We may say that the set of boys has MORE members than the set of girls.

- Have a set of cars and a set of pets ready for children to use on the magnetic board. You will need more cars.

Bob, will you please put the members of this set of cars on the left side of the magnetic board?

Charles, please put the members of this set of pets on the right side of the board.

Julie, will you pair the members of these two sets?

Is there a car for each pet? (Yes.)

Is there a pet for each car? (No.)

Does one set have more members than the other? (Yes.)

Which set has more members? (The set of cars.)

- Give each child two sets of objects that cannot be put into one-to-one correspondence. Ask each child to tell which of his sets had more members than the other.
- Have a box of small objects to distribute to the children. Without counting, quickly pick out a handful for each child. Have two children work together. (You may need to be the partner for one of the children.) Ask each group of two children to pair the members of their sets. Then ask questions such as these:

  Do your sets match? or, Are your sets equivalent?

  Does one set have more members than the other set?

  If it does, which set has more members?

- Put a set of bottle caps with many (20 to 30) members on the floor where all can see them. Place another set with a few (2 to 4) members on the floor near it. Ask the children which set has more members than the other. If the children choose the correct set, ask why they chose it. If a child responds, "It is bigger", substitute some very large blocks for the set with the few members and ask again which set has more members, and why.
- For some groups, there may be a need to include more examples of comparing sets which contain objects of different sizes (large blocks with pick-up sticks; large play trucks with small plastic cars or trucks) to help children concentrate on pairing the members of the sets rather than on the size of the objects in the sets.
- With some children you may wish to play a game in which they will have to compare imagined sets.

  Suppose we think about some things that are not in the room.

  First, think of a chicken. Now think of a dog.

  Can you make believe that you see them?

  Look at the legs of the chicken.

  Look at the legs of the dog.

  Which has more legs, the chicken or the dog?

  Next, think of the boots belonging to all the children in our room and the boots belonging to the children in your family.

  Which set of boots has more members?

If your children enjoy the game imagining sets, there are many other comparisons which may be used. Be sure to use only those with which the children are familiar.

If visualizing imagined sets seems too difficult, try another activity now but gradually try to increase the children's ability to use their imagination and to enjoy doing it.

- Play "Musical Chairs". Have each child place a chair in position for the game. Remind them that in playing the game a chair is removed each time the music stops so that the set of children participating will always contain more members than the set of chairs being used. Therefore, when the music stops there will be some child without a chair.

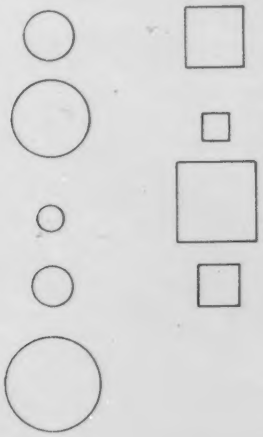
  Games of this type generally hold the children's interest and get quick responses that show their understanding of the concept.

- When displaying sets on a flannel board or magnetic board, sometimes place one set above or below the other, rather than to the right or left of the other. Children need to be able to compare sets when they are displayed in this way as well as when they are displayed in the other way.

FEWER THAN.

For some children the idea of "fewer members than" seems more difficult to grasp than the idea of "more members than". However, children's understanding of "more than" can be used to good advantage in developing an understanding of the related idea of "fewer than".

- Select one of the activities that was particularly successful when developing the idea of "more members than". Suppose, for instance, it was an activity in which members of sets were displayed on a flannel board, as:



After pairing members and reaching the conclusion that the set of circular shapes has more members than the set of square shapes, simply mention that

We may say this in another way. We may say that the set of square shapes has FEWER MEMBERS THAN the set of circular shapes.

This set (pointing to the circles) has MORE members than that set (pointing to the squares).

And that set (pointing to the squares) has FEWER members than this set (pointing to the circles).

- Use several other familiar pairing activities. In each instance, first have children identify the set with more members than the other set. Then have children identify the set with fewer members than the other set.

- In some instances, you will wish to concentrate only on the set with fewer members, and make no explicit mention of the set with more members. For instance:

Play musical chairs. This time emphasize that when the music stops, there are fewer chairs than children.

- Place a set of spools and a set of buttons, each with nearly the same number of members (8 to 10) on the floor. Ask the children which set has fewer members. There may be some disagreement. Ask

How can we be sure of which set has fewer members?

Ask a child to pair members of the set of spools with members of the set of buttons to find the answer.

- Place a pair of bags containing sets of familiar objects in various places around the room. (More groupings allow more participation for more children in a shorter space of time.) In each pair of bags you might put sets of objects such as: 8 pieces of doll clothing and 4 paint brushes; a box of sand and 4 blocks; a dozen sheets of paper and 1 pair of scissors; a drum and 10 small plastic cowboys; etc. Have a small disk to give to each child.

When we lift two bags, can we always tell which one has more things in it and which one has fewer things? (No.)

Today I have placed around the room several bags filled with sets of familiar objects. As you can see, these bags have been paired and they must stay together.

I will tell you which bags you will use.

Lift both bags and quickly guess which bag contains the set with FEWER members.

When you have decided, put a small disk by that bag and then come back to where you were sitting.

Remember I said "lift". Please do not shake the bags.

After the children are back in the group, have two children bring one set of bags back to the group where the bags can be opened and the sets compared. Also note each time which set was judged to contain fewer members. The members of most sets will not need to be paired, but if there is doubt, ask that a child pair the members to see which had fewer. Continue until all sets are compared. "You made some good guesses but some of the bags were "foolers", weren't they?"

- Have some children compare imagined sets.

Let's just THINK about some sets.

I'll tell you what I want you to "see" in your thinking.

Think of the wheels of a tricycle.

Now think of the wheels of a bicycle.

Which has fewer members, the set of wheels of the tricycle or the set of wheels of the bicycle?

Think of the wheels of a wagon.

Think of the wheels of a tricycle.

Which has FEWER members, the set of wheels of the wagon or the set of wheels of the tricycle?

Many other comparisons which are familiar to the children may be used.

ADDITIONAL ACTIVITIES

1. Prepare for games, dances, etc., which require teams with the same number of members on each team. Pair the members of the team to see if each team has as many members as the other or if one team has more members than or fewer members than the other.

2. Provide further experiences in pairing members of sets. On some occasions, be certain that there are more or fewer members in one set than in the other. On other occasions, there should be just as many members in one set as in the other.

3. Provide many opportunities for "Show and Tell" type activities to re-emphasize the concepts of as many as, fewer than and more than.

4. Use the pictures around the room to stress the concepts. We have as many pictures of birds as we have flowers. We have fewer pictures of houses than we have children. We have more pictures of animals than of trees, etc.

5. Some children may find dominoes helpful when used informally for practice in determining as many as, more than and fewer than.

6. If facilities are available, a cookie baking session may help reinforce the concepts of pairing.

   Did we bake a cookie for each child?

   Are there any cookies left over? (Yes.)

   What can we say about the members of the set of cookies and the members of the set of children in the class?

7. Have available a set of perception cards. Place a perception card on the flannel (magnetic) board. Let a child select another card from the set and look at it carefully. He must place it on the flannel board, to the left if it has fewer members, and to the right if it has more members than the card already on the board.

8. A very simple activity to illustrate the idea of pairing and as many as is to have each child take his name card and hold it in front of him.

   Is there a card for every child?

   Is there a child for every card?

   Are there as many members in the set of cards as there are in the set of children?

   (When there are absences, this activity may be used to show that there are more cards than children or fewer children than cards.)

Chapter 4

SUBSET OF A SET

OBJECTIVE: To introduce the idea of subset of a set.

VOCABULARY: Subset; (Review) set, member of a set.

MATERIALS: Variety of objects to use with flannel or magnetic board, several sheets of construction paper (each sheet a different color), toys, colored blocks, multi-colored beads, parquetry blocks, play dishes, dollhouse furniture, rhythm instruments, real fruit, teachers's "treasure box".

BACKGROUND NOTES:

In order to make the idea of subset meaningful to the children, it may be necessary for a while to use the expression "set within a set". The set must be identified first in each instance before discussing subsets of that given set. In speaking of a subset we must always have a reference set.

When the idea of subset is introduced, the children may tend to always select like objects as members of the subset. You will have to provide numerous opportunities for them to manipulate set materials and form various subsets so that they will understand that a subset may be any set within a given set. When identifying subsets of a set, make clear that since a subset is a set, a subset likewise may but need not consist of like members.

In view of this, avoid having the children develop the misconception that a subset is a subset because the members belong together for reasons based on size, color, use, etc. You may find that it will be effective to do more "showing" than "telling".

Although every set is a subset of itself, no explicit attention is given to this fact in the suggested activities. The emphasis is upon the idea of a proper subset of a set, but we simply speak of "subset" rather than "proper subset".

It is true that each individual member of a set is a subset of that set. However, in the early activities with subsets we suggest that you generally consider subsets that have at least two members. In this way children are less likely to confuse the idea of "member of a set" with the idea of "subset of a set".

ACTIVITIES TO HELP DEVELOP THIS CONCEPT:

Have the children seated around you so that all can see. Place on a table or rug a set of objects; e.g., a toy truck, a doll, a ball, and a book.

Karla, what have I placed on the table (rug)? Yes, we have a SET of toys.

Harry, will you name the members of our set of toys?

Hold up the doll and the ball.

Do these toys belong to our set of toys? (Yes.)

Put the doll and the ball back in place. Hold up the book, truck and doll.

Are these members of our set of toys? (Yes.)

Now, starting with the original set, place the doll and the ball together on the table (rug).

The doll and ball is a SET WITHIN our SET of toys.

Then, starting again with the original set, place the book, truck and doll together on the table (rug).

The book, truck and doll is a set within our set of toys.

Again, start with the original set of toys and clearly identify the doll and ball together.

The doll and the ball is a set within our set of toys. It is a SUBSET of our set of toys.

Start with the original set of toys and clearly identify the book, truck and doll together.

The book, the truck and the doll is a set within our set of toys. It is a SUBSET of our set of toys.

Ask one or more of the children to show some other subset(s) of the set of toys.

- Have a set of objects on the magnetic or flannel board; e.g., apples, stars, ducks, trees. Have the set identified. Move together a duck, a star and a tree.

  Do all these objects belong to the set that is on the board? (Yes.)

  Put those objects back in place and group together an apple, two stars, and a tree.

  Are all these objects members of the set on the board? (Yes.)

  Yes, we can say that they are a set within our set.

  Move all the ducks so they are together within a set.

  Are these ducks a set within the set that is on the board? (Yes.)

  Yes, we can say that the ducks are a SUBSET of the set on the board.

  Give several children an opportunity to form other subsets on the board.

- Place small dollhouse furniture on a table or on the rug. Ask a child to identify the set.

  Jane, will you touch the members of a subset that could be used in the kitchen?

  Leonard, will you touch the members of a subset that could be used in the living room?

  Continue until all the furniture has been touched.

  Is there someone who would choose a different subset for the living room?

  A different subset for the dining room?

- The girls may enjoy using the set of dishes from the dollhouse to form subsets. Showing and handling objects will aid the childrens' understanding of the idea of subset (set within a set).

- Assemble a set of toys that show the "Ways We Can Travel" -- cars, trucks, wagons, boats, planes, etc. After the set has been identified, ask a child to touch (without moving) the members of a subset he would like to work and play with during the day. Ask several other children to quickly touch the members of subsets of their choice.

  There is a set of yellow trucks on my desk. Is it a subset of this set of toys? (No, it is not. Its members do not belong to this set of toys.)

  Identify a set of toy boats someplace else in the room and ask questions similar to those above about it.

  Ask a child to quickly touch the members of a subset of our set of toys. See if other members of the class agree that his choice is a subset of the set of toys.

•This same type of activity might be carried on using objects from the teacher's "treasure box". As the children sit in a circle on the floor, open the box and quickly take out things.

The objects in the "treasure box" should have an element of surprise rather than being objects that the children usually see. Have more than one of some of the objects in the "treasure box". Ask individual children to touch the members of subsets of their choice, hoping they will take a mixed group.

Jim, in the lower right drawer of my desk there is a set of animal puppets.

Are the members of that set a subset of the set of things in the "treasure box"? (No. They don't belong to the set of things in the "treasure box".)

Have another child select a subset of the set of objects in the teacher's "treasure box".

•If classroom helpers are used, they may be thought of as members of the set of helpers. Identify the subsets of this set. (The subset of helpers who water the plants in the room; the subset of helpers who pass certain items at snack time, etc.)

•Discuss some family groups and see if the children can create subsets of members of the family. They may be interested to start with their own families.

Charles, who are the members of your family? (Grandmother, Mother, Father, Mary, Gloria and Bill).

Now name the members of a subset of your family.

Have one or two other children do this.

•Ask the children to recall a story about a family. Suppose they select "The Three Bears". You may ask,

Were the members of the family people or animals? (Animals.)

Name the members of the set of bears.

Name the members of a subset of the set of bears.

Would someone else name another subset of the set of bears?

You may find it necessary to refresh the children's memory by showing pictures from stories that have been read.

•Some child may have pets at home. You might ask him to tell the class about the members of his set of pets. After the members of the set are identified, have other children name the members of subsets of the set of pets.

- Have as your set for the day, several pieces of real fruit that are familiar to the children. You might choose an orange, a grapefruit, an apple, a pear, a banana, and a peach.

  Today, I want you to look very carefully at each member of our set. Then tell me as many things as you can about each member that would help us to identify it if we could feel but not see it. These things will be our clues.

  Is each member a subset of the set of fruit? (Yes.)

After the clues are identified, have the fruit passed around the circle so that each child can feel each piece of fruit. Then after either blindfolding a child or holding a newspaper between his eyes and his hands, place a piece of fruit in his hands.

Feel the fruit and tell us what subset you have in your hands.

Continue until many or all have an opportunity to so identify a subset of the set of fruit.

A carrot or potato might be put into the hands of a more alert child to see if he would recognize the difference and know that he was not holding a subset of the set he had talked about as the activity began. There might be many children in the class who would enjoy participating in this little "joke".

- On the flannel board, place a set that may include fruit, geometric figures, trees, flags, and animals. Ask a child to indicate a subset of fruit (which might include a red apple). Ask another to indicate a subset of all red objects (which might include the same red apple). Ask another to indicate a subset of fruit that grows on a tree (and the apple might again be included).

- Place several colored blocks (or crayons, sheets of construction paper, etc.) on a table. Have the set described. Invite the children to form various subsets (red blocks, yellow blocks, some red and some yellow blocks, etc.) Ask if the red blocks are members of the set of blocks, if the yellow blocks are members of the set of blocks, etc., to focus attention back on the original set.

  Refer to the buildings in a town as the set of buildings. Identify the subsets. (For example, houses, churches, hospitals, stores, schools, libraries, etc.)

- In addition to separating the class into subsets of boys and girls, you may separate it into subsets of children who wear glasses, boys wearing sweaters, girls wearing green, children with tie shoes, those who want to work in certain work groups, etc.

- Identify various subsets within a set of multi-colored beads or parquetry blocks.

- On the flannel board, arrange a set of felt or construction paper farm and zoo animals. After the set has been identified, ask one or two children to find specific sets.

  John, will you make a subset of the animals we might find on a farm?

  Mildred, please show us a subset of all the baby animals.

Next, ask individual children to form subsets and to tell what subsets they are forming. For example: a subset of animals that can go into the water (seal, turtle); a subset of animals that can climb a tree (a squirrel and a monkey); a subset of animals that have wings (chickens, ducks, birds).

- Without forcing things, take advantage of opportunities that arise day by day to reinforce the idea of subsets of a set.

Chapter 5

JOINING AND REMOVING

OBJECTIVES: To introduce the set operation of joining and its commutative property.
To introduce vocabulary and meaning for removing a subset from a set.
To introduce the idea of remaining set.

VOCABULARY: Join, remove, remaining, remaining set.

MATERIALS: A variety of sets of objects such as: small animals, blocks, "wheeled" toys, dolls, doll clothes, name cards, materials for flannel or magnetic board, yarn or string.

ACTIVITIES TO HELP DEVELOP THESE CONCEPTS:

JOINING

Have the children seated as usual in the classroom. Then you may wish to proceed as follows.

Girls, please stand.

Who are the members of this set? Yes, all of the girls in the room are members of the set of girls.

I want the members of the set of girls to come and sit here.

Now, I would like to have the boys stand.

Who are the members of the set of boys? Yes, all of the boys in the room are members of the set of boys.

Is Harry a member of the set? (Yes.)

Is Charles a member of the set? (Yes.)

Will the members of the set of boys JOIN the members of the set of girls?

Do we have a new set? (Yes, we have a set of girls and boys.)

Who are the members of our new set? (Each member of the set of girls and each member of the set of boys is a member of our new set.)

Is Roberta a member of the new set? (Yes.)

Is Jack a member? (Yes.)

When we join one set with another, we form a new set.

We started with the set of girls. Then the set of boys JOINED the set of girls. This formed our new set, the set of girls and boys.

What do you think would happen if we start with the set of boys?

Repeat the original activity, this time starting with the boys.

Is the new set that is now formed the same set that was formed when we started with the set of girls?

Yes, each member of the set of girls and each member of the set of boys is a member of the new set.

- Have ready a box of flannel or magnetic board objects near the board. Select a child to put up a set of objects. Ask someone to identify the members of the set. Have a second child put up another set of objects on the other side of the board. Let the children identify the members of this set. Now have a third child join the sets. Ask the class to identify the members of the new set.

Repeat the activity but start with the set selected last, then join the first set to it. Now, have the class identify the members of the new set. Ask,

Are the members of this new set the same as the members of the other new set?

Does it matter which set I start with when I am joining two sets? (No, the new set will be the same.)

When we join one set with another, we form a new set whose members are all of the members of each of the sets that were joined.

- During the daily activities, there will be many opportunities to have one set joined to another. For example:

1. one set of picture books joined to another set of picture books.
2. one set of toys joined to another set of toys.
3. one set of blocks joined to another set of blocks.
4. a set of sand shovels joined to a set of sand pails.
5. a set of brooms joined to a set of mops.
6. a set of new games joined to the set of old ones in the cupboard.
7. one set of children joined to another set of children for a given activity.
8. a set of children joined to a set of toys.

Be sure that they understand that the new set is the same no matter which set is used as the starting set.

•Have the sheep and cows identified as the sets to be used today.

You have had fun using these small wooden farm animals and the blocks.

Today, let's pretend that these blocks which I'll put on the table will form a fence around a pasture.

We'll put the sheep in the pasture.

Mr. Brown, the farmer, milked his cows this morning.

He is now taking them to the pasture to join the sheep.

Mike, will you please open the gate so that the cows can JOIN the sheep?

When we join one set with another, we form a new set.

What are the members of this new set? (Each member of the set of sheep and each member of the set of cows.)

•As the children are sitting in a circle, provide each with a set containing three or four members (plastic cars, small plastic or wooden animals, marbles, small dolls, doll clothes, blocks, etc.)

You each have a set of toys that you like.

Let's pretend you're going to a friend's home to play and are taking your set with you.

Ruth, will you choose someone with whom you would like to play, join your set to hers, and tell us what new set you and your friend now have?

Continue until all children are paired or only one child remains without a partner. If the latter happens, let him join his set with the sets of two other children. Name the new set.

•On the floor, make "a circus ring" using yarn, string, or rope. Have rubber or wooden zoo animals at hand (or mounted pictures of the animals if the objects are not available).

Let's go to the circus today. The lions will be the first animals to perform. Wade, will you be the lion-tamer and put your set of lions inside the ring?

Now that we've enjoyed their act, let's ask Glenn to put the tigers inside the ring to JOIN the lions in a new set.

In our unusual circus, the elephants perform an act with the lions and tigers.

Sharon is the elephant trainer who will JOIN her animals to the others in the ring.

Now we have a new act using a set of (lions, tigers, and elephants).

Does this set contain a subset of lions? (Yes.)

A subset of elephants? (Yes.) A subset of tigers? (Yes.)

Ask children to join other sets to the set of animals in the ring and identify the new set. After two or three sets are joined in the ring, ask to have them removed so that a new act may start.

REMOVING SUBSETS AND THE REMAINING SET.

On the flannel or magnetic board, place a set of ducks and birds. Have the children identify the set.

Susie, will you REMOVE the subset of birds?

Are all birds removed from the board? (Yes.)

Gay, what set is left on the board? (The set of ducks.)

We call this the REMAINING SET. (Point to the set of ducks on the board.)

When we REMOVE a subset from our starting set, the set that is left is the REMAINING SET.

Have the children answer such questions directed to these ideas:

Was each member of the set removed a member of our starting set? (Yes.)

Is each member of the remaining set a member of our starting set? (Yes.)

Was each member of our starting set either a member of the set removed or the remaining set? (Yes.)

Is the remaining set a subset of the starting set? (Yes, all members of the remaining set were members of the starting set.)

•Place on the magnetic or flannel board a set consisting of trees, flags, ducks, and rabbits. Have the set identified.

Mildred, please REMOVE A SUBSET containing all the flags, ducks and trees.

Now, what objects still remain on the board? (The rabbits.)

We call this set the REMAINING SET.

The remaining set is the subset which is left when we remove a subset from our starting set.

- Your children may enjoy playing store to reinforce the concepts of removing a subset and the remaining set. They may identify the set of items in the store. Then each child who buys something identifies the subset he purchases and removes it from the store. Have a child identify the members of the remaining set.

- Use imaginary sets.

  Think of a set of two girls.

  Now join to them a set of two new bicycles.

  Can you see the new set of girls and bicycles?

  Think of a set of Halloween witches.

  Now join a set of black cats to the set of witches.

  What do we have in the new set that we made? (Yes, witches and black cats.)

  Think of a set of white kittens and red balls.

  Remove the set of balls.

  Is just the set of kittens left? What do we call this set of kittens? (Yes, the remaining set.)

  Think of a set of red kites and green balloons.

  Remove the set of balloons.

  Which set remains? (Yes, the set of kites.) Is this set a subset of our starting set? (Yes.)

After giving several other examples, let the children suggest their own imagined sets. Ask them to think of joining one set to another or of removing a subset from a set.

ADDITIONAL ACTIVITIES

1. Have name cards for children placed face down on a table. Ask a child to pick out one card and tell and show the group the name on it.

   Sharon's name was picked out.

   She may make her choice of activity now and invite some others to JOIN her in this activity.

   The set will now include Sharon and (Ruth, Mark, and Lisa).

Have another child pick out a name. Continue until children and activities are paired. Emphasize the word "join" when asking the child to choose others to join him in the activity and have him name the members of each set involved in the joining.

2. Put the wheeled toys together in a group and ask to have the set described. Show the set of cars without moving it from the larger set.

   Is this set of cars a subset of these wheeled toys? (Yes.)

   Can we remove these cars from the set of toys? (Yes.)

   Move the car slightly away from the rest of the toys.

   What set do we have remaining here? (The other wheeled toys.)

   If you know what we call the set that is remaining here, come up one at a time and whisper the answer into my ear. (The remaining set.)

3. Using flannel board materials, tell a story about a pond in the woods where the animals come to drink.

   A set of ducks comes to the pond and a few minutes later is joined by a set of rabbits. (We now have a set of ducks and rabbits.)

   Soon a shy fawn quietly joins the ducks and the rabbits. (We now have a set of ducks, rabbits and a fawn.)

   Some cardinals come flying down to join the set that is already there. (Our set now consists of ducks, rabbits, a fawn, and cardinals.)

   Unknown to them, a hungry fox is lying in the deep grass at the water's edge.

   He greedily watches to see which member he can catch for his dinner, but before he can spring, the fawn smells him and bounds away, removing herself from the danger. (Remove fawn.)

   Her movements frightens the rabbits who quickly hop away to their homes, thus removing any chance the fox has for a rabbit dinner. (Remove rabbits.)

   The cardinals see the fox and fly away calling, "Fox, fox!", in time for the ducks to fly away with them. (Remove birds and ducks.)

   Sadly, the hungry fox looks to see what animal remains for his dinner.

   What set is remaining? (The empty set.)

4. Place a set of objects on the flannel or magnetic board. Have pupils identify the members of the set. Tell the children to close their eyes, then remove a member of the set. Have pupils open eyes and tell the member of the set that was removed. Have pupils identify the remaining set. The activities suggested may also be used to strengthen previously developed concepts. For example, you may ask such questions as:

   Which set has more members, the remaining set or the starting set?

   Which set has fewer members, the set that was removed or the starting set?

   This may be extended to the following: start with a set consisting of several apples and several oranges. When the children have their eyes closed, remove all the apples. Then have identified the set removed and the set remaining.

Chapter 6

PRE-MEASUREMENT CONCEPTS

OBJECTIVES: To become aware of differences in sizes of objects.
To learn to use appropriate words to describe such differences.

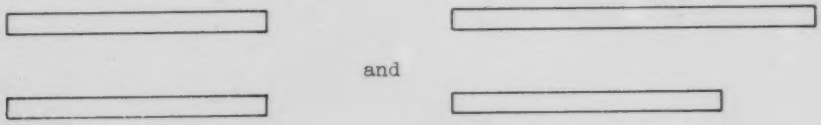
VOCABULARY: Longer, longest; shorter, shortest; taller, tallest; wider, widest; narrower, narrowest; larger, largest; bigger, biggest; smaller, smallest; thicker, thickest; heavier, heaviest; lighter, lighest; nearer, nearest; closer, closest; farther, fartherest.

MATERIALS: Sticks, some of which are different only with respect to their length, and others which differ in color, thickness, etc. as well as length; "nests" of boxes, barrels, blocks, etc.; rope or clothesline, yarn; balls; balloons; paper plates; cutouts of geometric regions, such that some can be placed entirely within the boundaries of others, without any overlapping; umbrellas.

ACTIVITIES TO HELP DEVELOP THESE CONCEPTS:

We shall make some detailed suggestions pertaining to the ability to compare lengths of objects. These suggestions then may serve as a guide in the development of other size comparisons.

- Have available a set of sticks that vary only in their length. Display two pairs of sticks, as shown below. For one pair, the sticks are the same length; for the other pair, they are not.



For the pair of sticks at the left, ask questions such as these:

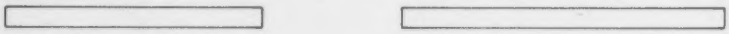
Is each stick just as long as the other stick? (Yes.)
Are the two sticks the same length? (Yes.)

Is one stick longer than the other stick? (No.)
Is one stick shorter than the other stick? (No.)

For the pair of sticks at the right, ask questions such as these:

Is each stick just as long as the other stick? (No.)
Are the two sticks the same length? (No.)
Is one stick longer than the other stick? (Yes.) Which one is the longer stick?
Is one stick shorter than the other stick? (Yes.) Which one is the shorter stick?

•Place two sticks "side-by-side", as below:



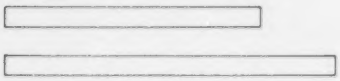
Then ask questions such as these:

Is each stick just as long as the other stick?
Are the two sticks the same length?
Is one stick longer than the other stick?
Is one stick shorter than the other stick?

If the sticks are not the same length, continue with these questions:

Which stick is the longer stick?
Which stick is the shorter stick?

If necessary, have the sticks rearranged as below to facilitate the comparison:



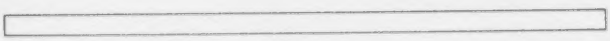
Continue with pairs of sticks in a variety of positions with respect to each other, using the preceding rearrangement whenever necessary. At times, use this form of rearrangment to verify the judgment made by children regarding the relative lengths of the sticks.

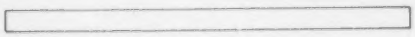
•Display one stick and, apart from it, a collection of several other sticks,--with at least one stick of the same length, one longer and one shorter. Then ask:

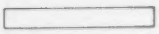
From the sticks here (pointing to the collection of other sticks), who can find a stick that is just as long as this one (pointing to the single "reference" stick)? Show me that the sticks are the same length. Is there another stick just as long as this one?

In a similar way, have children select and show each stick that is shorter than the reference stick, and each stick that is longer than the reference stick?

- Display three sticks as illustrated below: (The letters are used here simply to distinguish among the sticks in the discussion which follows.)

(a) 

(b) 

(c) 

First, compare stick (b) with stick (c):

Is this stick (point to b) longer than that stick (pointing to c)? (Yes.)

Next, compare stick (a) with stick (b):

Is this stick (point to a) longer than that stick (point to b)? (Yes.)

Is this stick (point to a) the longest of these sticks? (Yes.)

Again start with stick (b) and ask:

Is this stick (point to b) shorter than that stick (point to a)? (Yes.)

Is this stick (point to c) shorter than that stick (point to b)? (Yes.)

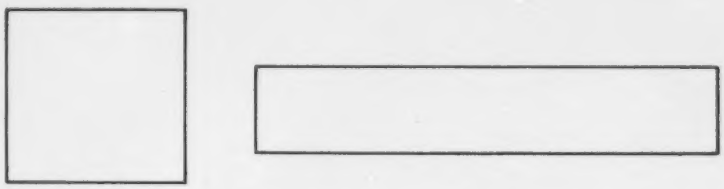
Is this stick (point to c) the shortest of these sticks? (Yes.)

- Present sets of three sticks in different positions relative to each other. By using questions similar to the above, have children identify the longest of the three sticks and the shortest of the three sticks.

- Provide experiences similar to the ones suggested thus far, but have sticks in vertical rather than horizontal positions. Lead to the fact that in such instances, we often speak of the taller or tallest thing rather than the longer or longest thing.

•Begin to vary attributes other than length when making length comparisons. For instance, use different colored sticks; use sticks with different widths or thicknesses; use things other than sticks,--pieces of string, rope, clothesline, yarn, etc.; use objects of different kind,--a pencil and an edge of a book, etc. In each of these instances, the property of length is compared and must be attended to in the presence of other properties that are irrelevant to length.

•You likely will find the preceding suggestions helpful as guidelines for developing other forms of comparison. When comparing sizes of objects in terms of ideas such as bigger, biggest, larger, largest, smaller, smallest, etc., it is essential that the materials used have the following property: for any two things being compared, one can clearly be contained in the other,--or we can clearly think of one as being contained in the other. To ask which of the two things below is "bigger", for instance, can only lead to trouble.



The following additional suggestions will reinforce ideas considered already, and also will extend children's understanding of other comparisons.

Display a set of objects on the flannel(magnetic) board. For example, you might use geometric shapes, either all of one kind or of various shapes. Tell the children to look very carefully at the objects. Permit them to handle the members of the set.

Look at the set of objects on the board.
Are they all the same size? (No.)
Are some members of this set SMALLER THAN other members? (Yes.)

Select some member of the set.

Look at the object I am holding.
Roberta, give me all of the members that are SMALLER THAN the one I chose.
Class do you agree?
Look at the objects that Roberta gave me.

Are some of them SMALLER THAN others? (Yes.)
Is there one member that is smaller than all of the other members? (Yes.)
Is it the SMALLEST member of the set? (Yes.)
In this set of objects we noticed that some members of the set are smaller than others.

Put all objects back on the board. Have various children select an object and then choose another child to select an object smaller than the one selected by the first child. Give them ample practice in identifying the smaller of two objects.

•At another time, have available several sets of three objects that fit one inside the other, for example, a "nest" of boxes or nursery blocks, objects from "The Learning Tower" or "Kittie in the Keg". Suppose you use a nest of boxes.

What do you think is inside this box?
Jack, will you remove the top from the box and show us what is inside?
(Yes, we have another box and it is smaller than the first one.)

Albert, will you open this box and describe what you find inside?
(Yes, this box is smaller than both of the others.)
We say that this box is the SMALLEST member of this set of boxes.

After the children have seen the boxes opened and noted that each is smaller than the one seen before, "scramble" them in a larger box or bag and have as many children as there are boxes pick out one without seeing the size. Allow a certain length of time for this "team" to put the boxes back together correctly. A three-minute egg timer might be used to designate the time allotment. Open the boxes again and place them in random order on your desk. Pick up the largest box in the set.

Mary, pick out the smallest box.
Is the box that Mary picked out the smallest one? (Yes.)
Is there a box left? (Yes.)
Is it smaller than mine? (Yes.)
Is it smaller than Mary's? (No.)
If we stacked these boxes inside each other, would we place Mary's box inside mine? (Yes.).
That is correct. Mary's box is the smallest box.
We would stack the smaller box first. Which box fits inside all the others? (Mary's box. The smallest box.)

Repeat this activity using other objects to reinforce the concept of small, smaller, smallest.

- Any of the above activities may be used to develop the concepts of larger and largest. Your children may be familiar with the terms bigger and biggest. Take advantage of this knowledge and relate them to terms you are introducing.
- A set of socks (a man's sock, a child's sock, an infant's sock or a doll's sock) may be helpful in developing the meaning and use of smaller, smallest, and larger, largest.
- Have the children find sets of objects in the room. Let them compare the objects according to size.
- Have children seated in a circle. Have available 5 geometric shapes of various sizes. As music is played the children pass the shapes. When the music stops, the five children holding the shapes, stand and arrange themselves in order according to size. Children answer questions such as:

  Who is holding the LARGEST shape?
  Is someone holding a shape SMALLER THAN the one Dorothy is holding?
  Who is holding the SMALLEST shape?

  Pin several pieces of yarn to the board. (Be sure to vary the length). Have the pupils examine the yarn. Then ask:

  Are all the pieces of yarn the same length? (No.)
  Are some pieces LONGER THAN others? (Yes.)
  Ronnie, will you point to one piece of yarn and then show us a piece that is longer than the one you selected?
  Is Ronnie right?
  Is there one piece of yarn that is LONGER THAN all of the others? (Yes.)
  Who would like to show that piece to the class? (Yes, this is the longest piece of yarn.)

  Have a similar activity using the concepts of shorter and shortest.

- Tie two different lengths of clothesline rope or heavy twine to the legs of chairs and coil the remaining rope by the chair. Have children estimate which is the longer rope by the size of the coil. Then ask two children to pick up the ends of the rope and walk in a designated path as far as the rope will permit them.

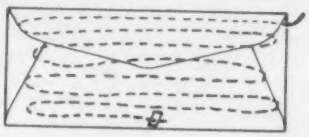
  Was the estimate right?

Recoil the ropes.

Bring out a third such coiled rope to see if the longest can be determined by looking at the size of the coil. Again check by having children walk with the ropes as far as they can go.

- Repeat the activity using different materials (different lengths of rope, twine, yarn) in each coil to have children note the difference in the thickness of the material used. See if they will take this difference into consideration when estimating which coil will be the longest and which will be the shortest. If the interest is sufficiently high, you may use this activity to introduce the idea of thicker and thickest.

- Another activity can be carried out using white yarn and white envelopes. Cut the yarn in different lengths having some as long as other lengths, some shorter than and some longer than the others. Scotch tape one end of each piece of the yarn in separate long business envelopes. Loop the yarn back and forth in the envelope, leaving the free end hanging slightly out.



Again designate the path the yarn is to be carried, to see who has the shortest or the longest pieces of yarn and which pieces are the same length. A child may be used to hold an envelope while another walks with the yarn. If you have a rug, pin the yarn to it when the length is fully out of the envelope so the differences in lengths can be seen easily by all.

This same type of activity may be continued at another time by using a bright or dark yarn in a thin envelope so that a clue is obtained by being able to slightly see the number of loops or how closely the loops are arranged in the envelope.

- Display on the board several strips of colored ribbon, tape or paper. Be sure that all strips are equal in length, but all differ in width.

Notice the strips on the board.
Are some of them longer than others? (No.)
Are some of them shorter than others? (No, they are all the same length.)
Are all of the strips exactly the same? (No, some are wider than others; some are narrower than others.)
Is there a strip that is WIDER than all of the others? (Yes.)
Mary, please show me the widest strip.
Is there a strip that is NARROWER than all of the others?
George, please show me the NARROWEST strip.

Other opportunities for the understanding of the concepts of narrower (narrowest) and wider (widest) can be found in comparing: the widths of rickrack on dresses, and widths of the stripes in boys' T shirts and girls' dresses, the widths of the turned up cuffs on the boys' jeans, the width of ribbons worn by the girls, etc.

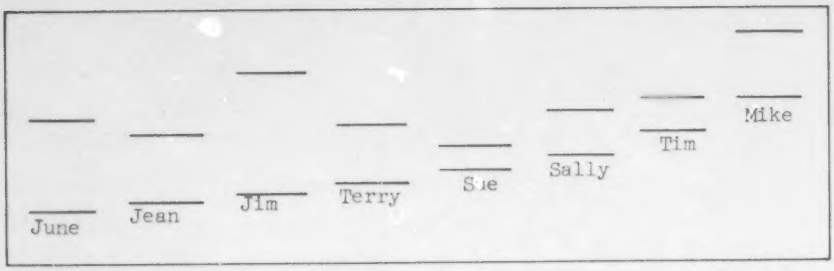
Identify five children as members of the set for discussion today. Have them arrange themselves according to height, starting with the shortest child. Then ask the class questions similar to the following:

Is Roberta TALLER than Amelia?
Is Amelia taller than Grace?
Who is the TALLEST member of the set?
Who is the SHORTEST member?
Is the tallest child, taller than every other child in this set? (Yes.)
Is the SHORTEST child SHORTER THAN every other child in this set? (Yes.)

- Have all of the girls arrange themselves according to height. The discussion that follows should include information concerning the shortest girl, the tallest girl, girls taller than or shorter than certain other girls, etc.

Try a similar activity using the boys in the class.

- With the children's help, arrange the class according to height from the shortest to the tallest. Using a roll of wrapping paper tacked to a bulletin board (if the space is narrow unroll just enough for use with one or two children at a time) measure each child, mark his height, and write his name under the mark. After the children have had time to compare the heights as they wish, roll the paper and put it away until the last week or two of school. Then again mark the heights stressing how much taller each child is than he was at the time of the first measurement.



- Have a child build a tower using the blocks. Then have several other children build towers that are taller than the previous one. At the end of the activity have pupils point to the tallest tower.

- Some rainy day when a supply of umbrellas is available, select a few of various heights. Have the pupils stand them up and decide which is the tallest umbrella. Let the owners identify their umbrellas. The discussion may be more interesting when the children can say "Mary's umbrella is taller than Karla's umbrella" or "Gloria has the tallest umbrella."

- Members of the family may be discussed to reinforce the concepts of shorter (shortest) and taller (tallest). For example:

  Father is the TALLEST member of the family.
  My brother is TALLER THAN my mother.
  My sister is the SHORTEST MEMBER of the family.
  I am SHORTER than my mother.

- The health program in some schools includes finding the heights and weights of the children. You may take advantage of this activity to reinforce the comparison words for tall and short and to introduce the comparison words for heavy and light. If you introduce these words, provide some other opportunity for the children to use them. Place objects such as blocks, plastic toys and sheets of paper in separate bags. Have a child lift each bag and decide which bag is lightest or heaviest. Have a child select any two bags and tell the class which bag is heavier or lighter than the other.

  A variation of this activity is to have the pupils select several objects of various shapes around the room as members of the set. First have the children guess which is heavier or lighter by sight. Select a child to feel the objects and arrange them from lightest to heaviest. Have another child feel the objects and see if he agrees with the arrangement.

ADDITIONAL ACTIVITIES

1. Unless you have a child in the class who has been made quite sensitive to his long or short feet, comparing the length of hands, feet, shoes, boots, and mittens brings meaning to the words shorter than, shortest, longer than and longest.

   If there are several pairs of boots that look much alike and do not have distinguishing marks on them, bring them to the group.

   > I want you all to sit with your legs out in front of you. Let's look at the length of this pair of boots and then at the length of all the shoes in our room.

   Hold the boots so that the children's attention is focused on the foot of the boots rather than the leg of them.

   > See if you can decide who might be able to comfortably wear these boots.
   > Mildred, will you please take these boots to one person whom you think they might fit?
   > Good, is there another person in our group who might wear these boots?
   > Jerry, can you find someone who has shoes that are shorter than these boots?
   > Sara, can you find someone who has shoes that are longer than these boots?

   A similar activity might be carried on using mittens.

2. On some other occasion use the children's boots, but include the teacher's boots in the set. This time, use one boot from each pair and stand them up on the floor or a table. Look at the leg of the boots and have children arrange them according to height starting with either the shortest or tallest boot.

3. You may wish to use a variation of this activity to review set language with your children. Make two equivalent sets of boots and place the members in a random order at two different locations in the room. Ask a set of children to join each set of boots. Have one set of children arrange the set of boots in order beginning with the tallest one. Have the other set of children arrange the boots in order by starting with the shortest boot. Give the children time

to look carefully at the two arrangements and decide whether or not they agree with the work of classmates. You may ask:

Are there MORE members in one set THAN in the other? (No.)
Are there FEWER members in one set? (No.)
Are there AS MANY members in one set AS there are in the other set? (Yes.)
Then what do you know about these sets of boots? (They are equivalent.)
How can you show this? Yes, you could pair the boots in this set (indicate) with the boots in this set (indicate) and see if there are any boots left.
Karla, will you come up and PAIR the MEMBERS of these sets?
Do the sets match? (Yes.)
Are they equivalent? (Yes.)

4. "I'm Thinking Of . . ."

I'm thinking of something that is bigger than I am.
I'm thinking of a horse.
Can you tell me something that is bigger than you are?
Now I'm thinking of something that is larger than a horse.
I'm thinking of an elephant.
Can you think of something that is larger than the thing you named before?
I'm thinking of something that is smaller than I am.
I'm thinking of a kitten.
Can you tell me something that is smaller than you are?
I'm thinking of something that is smaller than the kitten.
I'm thinking of a piece of pie.
Can you think of something that is smaller than the thing you named before?
What is the smallest thing that you can think of?
Is this smaller than everything you named before?

5. This type of thinking about relationships as to size can be used with wider and narrower; taller and shorter; etc.

6. Turn 5 or 6 paper plates of graduated sizes upside down and arrange on a tray so that only the largest shows.

What do you see on the tray? (A paper plate.)
I'll remove the plate. Now what do you see? (Another plate.)
I'll remove it. Now what do you see? (Another plate!)

Continue until all plates have been removed.

How was I able to arrange these plates so that you saw only one plate at a time?

Many children will be able to tell you that each plate was just a little smaller than the one they had seen before. To help some see more clearly, you may want to ask a child to demonstrate by holding his hands just far enough apart for you to slip the smallest plate between them.

Show me with your hands what John will have to do before I can slip this plate that is just a little larger between his hands.
This plate is still larger than the last one.
Show me how your hands will have to move now.

Continue until all plates have been used.

Scatter the plates around in a small space and have a child see how quickly he can arrange them as they were.

Then have them scattered and ask a child to turn them over and arrange them with the largest on bottom.

7. Paper plates of graduated sizes, "nests" of boxes or barrels, graduated sized beads, etc., might be made available for children to arrange and rearrange during the work period. Have sets of rectangular paper plates as well as circular ones.

8. Give each child a balloon.

I have a balloon for each of you today.
Let's stretch them before we start blowing so that we can blow them up more easily.
You may blow them as large as you wish after we finish our game, but for now I will want you to stop blowing when I say, "Stop!"
Blow.

Have children blow until a balloon is about 3" in diameter.

Stop!
Is your balloon larger than it was when we started? (Yes.)
Blow.
Stop!
Is your balloon the largest it has been since you started blowing? (Yes.)
Now you may blow your balloon as large as you wish.
When you have it that large, you can fasten it with a rubber band you'll find on the workbench.
I'll give you help if you need it.

9. Before the children come into the room or while they are busy elsewhere place five or six umbrellas on a small table with the longest umbrella to the front and the rest arranged in graduated size to the smallest umbrella. (e.g., man's umbrella, woman's smaller umbrella, child's umbrella, a Japanese toy sun umbrella, and a small party-favor umbrella.

What do you see? (An umbrella.)
Show me how far apart you think you'd need to stretch your hands to be able to reach from the point of the umbrella to its handle.
Chris, will you come and see if your hands are stretched out enough to reach that far?

As he comes to the table, whisper to him to say nothing about what he sees behind the big umbrella, but to pick up the umbrella and stand in front of the others.

Chris's hands were just about the right distance apart, weren't they? Chris, you may put the umbrella on the chair. What do you see now? (Yes, you see another umbrella.)

Continue with the remaining umbrellas in the same manner stressing that each is smaller than the one they saw before and that their hands are getting closer and closer together.

10. At another time, the same umbrellas might be used to demonstrate width. Open all the umbrellas and line them up according to graduated size with the handles facing away from the children.

Is each umbrella just as wide as the one next to it? (No.)
Jean, will you please go and sit behind the widest umbrella.
Did she choose the right one?
Can you see her? (No.)
Jack, will you sit behind the narrowest umbrella.
Did he choose the right one?
Can you see him? (Yes.)
Carol, is there another one that is wide enough for you to sit behind so that we can't see you. (Yes.)
Can you see her? (No.)

Close the umbrellas and place them at random on the floor.

Tom, if we held the umbrellas up straight with the points on the floor, which one would be the shortest? Please show us.
Sara, which one do you think would be next in size?
Stand your umbrella up beside Tom's.
Is it taller than Tom's umbrella?

Continue in this manner until all are used. Speak of the umbrellas as being taller than and tallest.

11. Another way to use the sense of feel to reinforce the concept of larger than or smaller than is with various sizes of balls, going from a small jacks ball up to a large beach ball. (Jacks ball, golf ball, a slightly larger rubber ball, tennis ball, etc.)

Hold this jacks ball tightly in your hand. (Palm downward.)
Can you see the ball? (No.)
Hold the golf ball in the same way.
Can you see the ball? (Yes, just a little.)
Can you still curl your fingers around the ball and hold it comfortably in one hand? (Yes.)
Let's use this rubber ball next.
Can you hold it in one hand with your fingers curled around it. (Yes.)
Has your hand spread farther out as you hold this ball? (Yes.)
Is this ball larger than the others? (Yes.)

Continue until the child has handled all the balls. When he comes to the first ball he can't hold in one hand (palm downward so that the ball isn't just balanced on the hand, ask him to put the ball between his hands. Help him to get the feel of largeness by the feel of stretching

as he handles larger and larger balls. Immediately after he has handled the largest, give him the smallest so that he may feel the sharp contrast.

12. The classroom equipment and the materials brought into the room by the children offer excellent opportunities for the comparison of length, width, and size.

My jumping rope is longer than yours.
I have the biggest ball.
You aren't as tall as I am.
I want a piece of paper the same size as Tim's.

These and many other statements or questions can lead naturally into a situation calling for an on-the-spot comparison.

Chapter 7

ORDERING OF SETS

OBJECTIVE: To introduce the idea of ordering of sets by using the concepts of more than and fewer than.

VOCABULARY: Order, between, (Review) more than, fewer than, right, left, equivalent.

MATERIALS: Small objects, paper bags, perception cards, chart holder.

ACTIVITIES TO HELP DEVELOP THIS CONCEPT:

Select a set of five girls and a set of five boys to serve as helpers today. Have them stand before the class. Let the class identify the members of each set.

Are there as many members in the set of girls as there are in the set of boys?
How can you find out? (Pair the members of the sets to see if the sets match.)
May I say that the sets are equivalent? (Yes.)
Ruth and Jean, please take your seats.
Let's name the members of the set of girls now standing here.
Now, are the sets here at the front of the room equivalent? (No, there are more boys.)
The set of boys has more members. Which set has fewer members? (The set of girls has fewer members.)
Is Karla a member of the set with more members? (No.)
Is Harry a member of the set with more members? (Yes.)
Is there any boy who is a member of the set with fewer members? (No, only girls beong to the set with fewer members.)
Will the children who belong to the set with fewer members please stand here (designating a location toward the left side of the front wall).

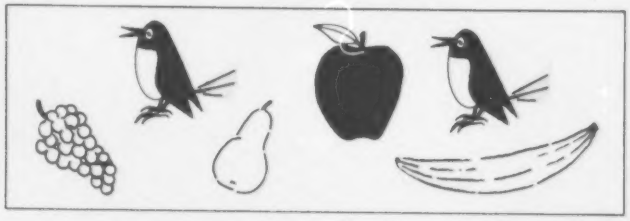
Will the children who belong to the set with more members please stand here (designating a location to the right of the previous one). (Pointing to the set at the left).

Is the first set the one with fewer members or the one with more members? (Fewer) (Pointing to the set at the right).

Does the next set have more members than the first (other) one? (Yes.)

Have the children sit down. Direct their attention to the flannel or magnetic board.

Place two sets that are not equivalent on the magnetic board. Let one set consist of fruit and the other of birds.



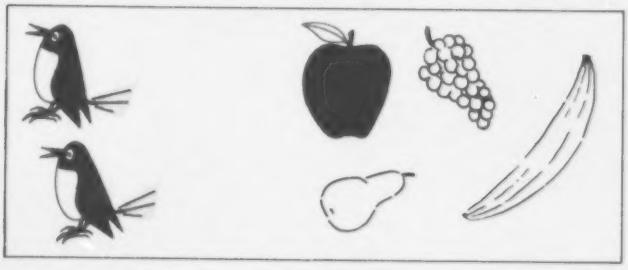
Let the children assist you in pairing the members of the set of fruit with those of the set of birds to determine whether or not the sets are equivalent.

Do the sets match? (No.)

Which set has fewer members? (The set of birds.)

Which set has more members? (The set of fruit.)

Let us arrange the set of birds here in this portion of the magnetic board and the set of fruit here. (See drawing for the arrangement.)

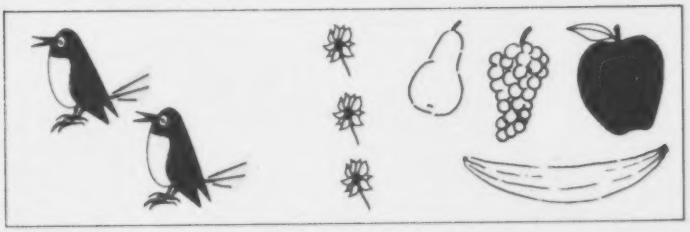


Select a set of flowers which has more members than the set of birds and fewer members than the set of fruit. Select a child to demonstrate by pairing members of the sets whether the set of flowers has more or fewer members than the set of birds.

Are there more flowers than birds? (Yes.)
Does the set of flowers have more members than the set of birds? (Yes.)
Are there fewer birds than flowers? (Yes.)
Does the set of birds have fewer members than the set of flowers? (Yes.)

Select another child to demonstrate whether the set of flowers has more or fewer members than the set of fruit.

Are there more flowers than pieces of fruit? (No.)
Does the set of flowers have more members than the set of fruit? (No.)
Are there fewer flowers than pieces of fruit? (Yes.)
Does the set of flowers have fewer members than the set of fruit? (Yes.)
Are there more pieces of fruit than flowers? (Yes.)
Does the set of fruit have more members than the set of flowers? (Yes.)
Let us put the flowers BETWEEN the birds and the fruit.



Are there more flowers than birds? (Yes.)
Are there more pieces of fruit than flowers? (Yes.)
Are there fewer pieces of fruit than flowers? (No.)
Are there fewer birds than flowers? (Yes.)
We have put the three sets IN ORDER The set of birds (pointing) is first. The set of flowers is next.
There are more flowers than birds. The set of fruit is last. There are more pieces of fruit than flowers.

Are there more pieces of fruit than birds? (Yes.)
Are there fewer birds than pieces of fruit? (Yes.)

- In the preceding activity, the third set came between the other two sets. Use materials of your own choosing and proceed in a similar way to illustrate instances in which the third set has:

  fewer members than either of the first two sets used;
  more members than either of the first two sets used.

If needed, take advantage of this opportunity to reinforce children's ability to distinguish among the set on the left, the set on the right, and the set between; and to refer to one set as being to the left or right of another set.

- Prepare in advance 3 sets of objects for each child. (For example, 5 paste sticks, 4 buttons and 3 blocks.) Have no more than five members in any set at this time. Give each child a container with a variety of objects.

  You have a set of paste sticks, a set of buttons, and a set of blocks.
  Place the members of the set of buttons on the table in front of you.
  (If other materials are used, start with whichever set will be between the other two, in order.)
  Now choose one of the other sets and pair its members with those of the first set.
  Does this set have more members than there are in your first set?
  Or does it have fewer members than your first set?
  (Answers will vary depending upon the choice of the set.)
  If it has more members, place them to the RIGHT of the members of your first set.
  If it has fewer members, place them to the LEFT of the members of your first set.

Since each child has the same materials, it should be easy for you to look around the room and see the arrangement of objects on their tables.

  Now, pair the members of your last set with those of your first set.
  Does this set have more or fewer members than your first set?
  If it has more members, place them to the right of the members of your first set.
  If it has fewer members, place them to the left of the members of your first set.

You have put the sets in ORDER.

One set is between the others. It has more members than the set at the left. The one between has fewer members than the set at the right.

- Have the children return all objects to the containers. Now lead the children through a procedure similar to the preceding one, but this time have them start with the set of blocks. Finally, go through the procedure again, this time starting with the set of paste sticks. Each of these may result in problems of ordering that were not encountered in the first instance, when the third set is compared with each of the other two. Some children will need considerable help and repeated experiences of this nature before they seem to grasp the essence of the ordering process.

- Repeat this kind of activity, but give the children sets of objects that vary in number of members from those of their neighbors. For example, one child may have 2 beads, 3 buttons, and 4 paste sticks, while another might have 4 beads, 2 paste sticks and 5 buttons.

- Give children different sets of objects and have each compare the number of members in this set with those of his neighbors, possibly ordering the sets by rearrangement.

- Place four sets of objects ranging from one to four members on the flannel or magnetic board. Do not have the sets arranged in order; for example,

Look at the members of the sets on the board.
Let us see if the sets are in order.
If they are not, I want you to help put them in order.
Look at the stars, here at the left (pointing).
Does the set of circular shapes have fewer members than the set of stars? (If necessary, have a child pair members of the sets to see that the answer is No.)

Does the set of square shapes have fewer members than the set of stars? (As before, No.)

Does the set of triangular shapes have fewer members than the set of stars? (Yes.)

Then which set shall we put first, here at the left? (The set of triangular shapes.)

Put the triangular cutout at the left of the two stars, such as:

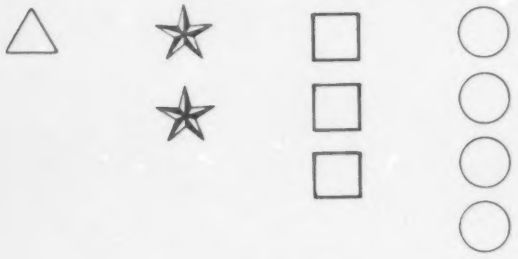
Let us see if the sets now are in order.

Does the set of triangular shapes have fewer members than the set of stars? (Yes.)

Does the set of stars have fewer members than the set of circular shapes? (Yes.)

Does the set of circular shapes have fewer members than the set of square shapes? (No.)

What can we do to put the sets in order? (Put the set of square shapes between the set of stars and the set of circular shapes, like this:



Have the children help you verify that: going from left to right, each set has fewer members than the one following it; and going from right to left, each set has more members than the one next to it.)

- Provide similar experiences in ordering other sets of objects. With sets presented in random order, use sets having 2, 3, 4 and 5 members; sets having 1, 2, 3 and 5 members; sets having 1, 3, 4 and 5 members; and sets having 1, 2, 3, 4 and 5 members. (The number of members in each set is given here only for convenience in communicating with you. The number of members in a set is not to be mentioned when working with children on ordering the sets.)

- Have ready five paper bags each containing a different number of familiar objects (from 1 through 5). Place the bags at random on the floor so that neither you nor the children know which bag holds a particular set. With all children facing a low table, or other area easily seen, pick a bag and place its contents in the center of the table. Ask a child to choose another bag, remove the set, and determine whether or not it has more or fewer members than yours. If his set has more members, have him put it on the right of the set on the table. You may wish to put these objects on colored paper or cards to help children identify the members of each set. If it has fewer members, place it to the left. Be sure that all children are sitting so that the directions, left and right, are correct from their viewpoint. Put the objects back in the bags and repeat the procedure several times. If a child cannot determine whether the set has more members than or fewer than the set on the table, ask him to check by pairing the members of the sets. After the first set selected is placed to the right or to the left of the starting set, have another child select another. It may go between the first two or to the left or to the right of the other sets. This activity may be used to order any three or four of the five sets, or to order all of them.

- Prepare perception cards* for sets having 1 through 5 members. Place the cards face down in a box. Pick one card at random to put into the center of the chart holder. Ask a child to pick out another card and determine whether the set on his card has more or fewer members than are in your set. If his set has fewer members, have that card placed on the left of your card. If it has more members, ask that it be placed to the right. Again, check to see that all children are sitting so that the directions, left and right, are correct from their viewpoint.

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* Suggestions for making perception cards may be found in the back of this book. The section is called Perception Cards.

- Using the perception cards, pick a card from the box for the beginning of the game. Ask a child to pick another card and without looking at it place it either to the right or left of the first. This child will then ask another child if he has placed it correctly. If he did not, the second child will tell him why it is not correct and move the card to its right position. The second child then removes all cards and starts the game with a new card picked at random. When the second child does not know if the set has more than or fewer members, ask him to check by pairing the members of the sets. Again, arrangements can be made by continuing to draw cards without replacements as was suggested for the objects in paper bags.

Chapter 8

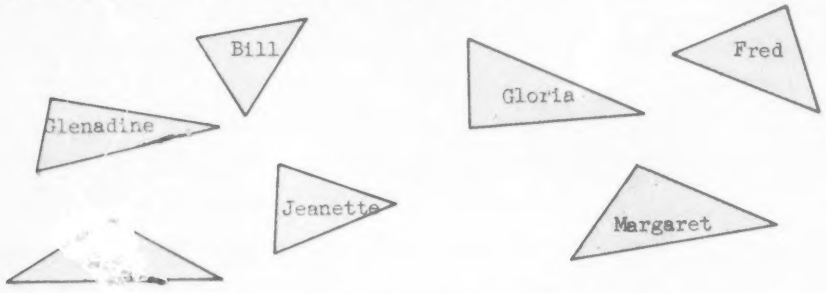
ADDITIONAL ACTIVITIES FOR DEVELOPING CERTAIN IDEAS USED IN MATHEMATICS

OBJECTIVE: To provide more opportunities for children to increase their understanding of certain concepts used in mathematics.

VOCABULARY: Above, around, beside, over, behind, top, position, lower, upper, middle, center, same, different, (Review) between, inside, on, largest, smallest.

MATERIALS: Colored construction paper, heavy cardboard; objects for magnetic or flannel board; felt; string; ball; chairs, tables; blocks; books, other objects that would normally be in the classroom.

At times when you want the children to sit in designated places their names can be written on colored paper cut-outs of various geometric regions. Use regions that have the same shape but differ in sizes and the positions in which they are placed. You may have the children identify the particular shape being used. For example, triangular regions might look like these.



Some other time use rectangular and circular regions in the same way. The regions should vary in colors as well as sizes.

- Have geometric relay. Divide the class into three or four teams. Seat (or stand) teams in rows Show a geometric figure, allow the first child on each team a minute or two to find another object shaped like it somewhere in the room. Ring a bell, and as soon as the first child is successful, he sits at the back of his row; then show another figure for the new leaders of each row

to find. This continues until each child has had a turn. The team whose members were able to find all or most of the shapes in the allotted time wins.

- Have cut-outs of various regions made with the name of a child written on each.

Place the cut-outs on the floor or a nearby table.

> Today, we will use a different way of deciding where children will work.
>
> As you may have noticed, there is a triangular shape by the big blocks, a rectangular one by the art materials, a circular shape by the small blocks, etc.
>
> As the cut-out with your name is turned over, please go quietly and stand in the place marked by a shape that is like yours.

This is a good way of seeing that some children work with materials that they seldom choose. At a later time these same cut-outs can be used to get the children working in yet another activity. Change children's names on regions so a child is not always identified with the same shape.

- Provide puzzle boxes of geometric figures. Make heavy cardboard cut-outs of rectangular regions (including square regions), triangular regions, and circular regions of various sizes. Cut those irregularly into two, three, or even four pieces to be fitted together to complete the figure.

- These pieces could also be used as a game to find who would work with the different types of equipment during the free activity period. After each child has taken a piece from the box, designate one child as the "key" man for each figure. He stands in one place holding his piece so that it can be seen easily. Every other child sees how quickly he can find where his piece of the puzzle fits. He has then found the children with whom he will work. The various groups will then work where they find another figure to match the one they have formed.

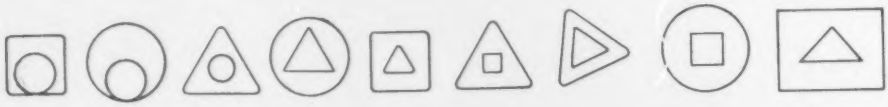
- Put cut-outs of regions on magnetic (flannel) board. Ask a child to place a piece of yarn on the board and describe its position in relation to the cut-outs.

> The yarn is BETWEEN the circle and the square.

It is ABOVE the rectangle.

It is AROUND the circle and the triangle.

- Use perception cards, starched string, wire, yarn, or construction paper to make visual aids such as the following:



Display only a few such figures at a time. Many questions could be asked, such as:

In which one do we see a TRIANGLE inside a square?
In which do we see two figures of the SAME shape but of different sizes?

- Make large cut-out felt or paper regions of various shapes. Then, cut out smaller regions from the inside. Each small region should have a shape different from the larger region from which it was cut. These can be used to fit into each other. Your figures may look like this.

Directions similar to these may be used.

Find the triangular shape that fits inside a rectangular shape.

Which square shape fits around a green circular shape?

Which circular shape will fit inside a triangular shape?

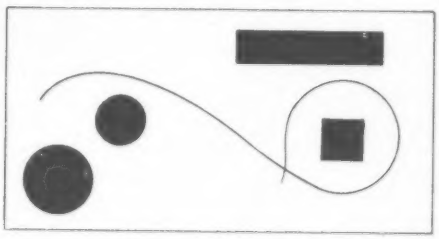
- A variation of the activity above is to make two figures, one that can be placed inside or around each shape and one that will just fit. The children may be requested to choose which blue triangular shape will fit around a particular circular shape or which yellow circular region will fit inside the red rectangular shape. This will require finer discrimination and give more challenge to some children who may be ready for a more difficult task.

- Make triangles, circles, and rectangles in graduated sizes as shown below. Wire, starched string or styrofoam may be used. Be sure that the shapes are sufficiently different in size so that one is contained wholly within the interior of another.

Using the above figures, the children might be directed to:

Place the LARGEST triangle on the largest circle.
Put the SMALLEST triangle inside the largest square.
Arrange the largest triangle between the smallest circle and the smallest square.

- Create collages using your cut-outs of regions.



Some questions about the collage might be:

What figure has the string around it?
The string is between what figures?

There are many opportunities to introduce and reinforce concepts of position or location, both in the room and on the playground. In your routine activities, you can use these words so that your children will get used to hearing them and attaching meaning to them.

Will you sit BETWEEN Carol and Jack?
I see some small blocks on the floor BETWEEN the workbenches.
Mary, will you stand BESIDE me for this game?
In "Pop Goes the Weasel", the "weasel" pops BETWEEN the two partners.

- Play "follow the leader" games. Directions can be used during the rhythm period to vary a marching activity. The "leader" must be alert to hear and respond to the direction and those behind him must follow. Change leaders often.

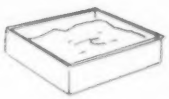
  Walk BETWEEN the tables.
  Walk BEHIND the green chair.
  Jump OVER the block.

  Seat four children at a table, e.g.,

Jean	Sara
Mike	Jack

  If I serve Mike first, can I serve Jean without going past another child? (Yes.)
  If I went the other way, who would be BETWEEN Mike and Jean? (Jack and Sara.)

Slide


Sandbox

Seesaw

- These, or similar activities may be more successful when the class is on the playground. However, you may use objects to make a representation of the playground equipment in the room.

  When Jim goes down the slide, does he have to go past the sandbox to get to the seesaw? (Yes.)
  Is the sandbox BETWEEN the slide and the seesaw? (Yes.)

Slide

Swings

Garden

When Jean goes down the slide, does she have to go past the garden to get to the swings? (No.)
Is the garden BETWEEN the slide and the swings? (No.)
Are the swings BETWEEN the slide and the garden? (Yes.)

- Have some objects close at hand that can be used in conjunction with your directions to the children.

Jim, put this car BETWEEN John and Tim.
Tom, put this paper in the LOWER, RIGHT-HAND drawer.
Julia, hold the book ABOVE Carol's head.
Now put it ON her head.

- Have a child (Jerry) stand several feet away from you and bounce a ball to him. Put another child (Joe) between you and the first child.

Can I bounce the ball to Jerry now? (No, Joe is between you and Jerry.)

- Some children will need more than incidental learning experiences to help them understand many of these concepts. "Place" games may be of help to them. These games may emphasize such positions as: above, below, behind, beside, top, bottom, upper, lower, corner, center, middle, over, and under.

Chapter 9

USING NUMBERS WITH SETS

OBJECTIVE: To help children identify the number of members in certain sets.

VOCABULARY: One, two, three, four, five.

MATERIALS: Set objects, materials for magnetic or flannel board, perception cards, card holder.

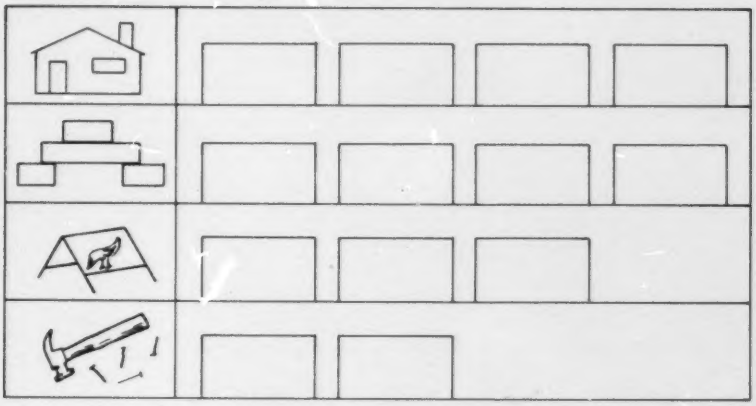
BACKGROUND NOTE:

The sets to be identified will be limited to no more than 5 members. The suggestions in this section of the commentary are designed to provide meaningful number associations as background for future systematic development of number ideas. There are many opportunities in the daily program for the child to recognize without counting, the number of members in sets with no more than five members. Counting *per se* is not considered at this stage of development.

ACTIVITIES TO HELP DEVELOP CONCEPTS

NUMBER OF MEMBERS IN SELECTED SETS

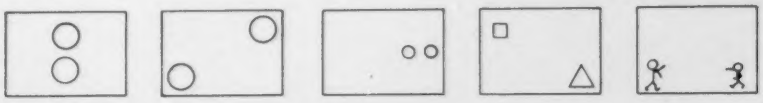
In some kindergartens, certain work areas are limited to 4 children at a time. A child soon is able to see at a glance whether or not there is room for him on the big blocks, in the dollhouse, at the workbench, etc.



Using a card holder, put in pictures showing the various work centers. As each child chooses his work for the day, he will put his small name card to the right of the pictured activity. The child can see at a glance whether or not there is room for him at that activity. This provides another opportunity for developing the concepts of "twoness", "threeness", etc.

- Singing in duets, trios, and quartets helps him to recognize the number of members in such sets.
- Six children may sit at each table at lunch time, three on a side.
- Arrange in various ways on the floor, 3 large blocks, 3 marbles, 3 balls, 3 paint brushes, and 3 small toys. Help the child to see that the arrangement, or the size of the objects did not change the fact that each set had "three" members. Similar arrangements may be made with materials on the magnetic board.
- Use this same type of activity with sets containing 2, 4, and 5 objects.
- There may be 4 pictures on a bulletin board which the children recognize as 2 sets of 2, or a set of 4 .

- Perception cards might be used, e.g.,



•Name cards may be used to designate where each child is to sit as he enters the group. The number of children absent for the day will be the same as the number of the cards that are not paired with children.

Cut-outs of different shaped regions might be used in the same manner.

PAIRING WITH AN IMAGINED SET

Before the children come in, place many small set materials around the room. Have materials that you can use to arrange in sets of 2, 3, 4, or 5. It may be best to use a variety of objects in the arrangements to reinforce the idea that sets may have dissimilar members. Tell the children that we will play a game of "Let's Pretend" today.

I will make believe I have certain sets and I will ask you to look around the room to find sets that match mine.

I have a make believe set whose members are a car, a cowboy and a teddy bear.
I would like to have you find a set that has as many members as my set.

I have a set containing a book, a ball, a top, and a doll.
Find a set that has the same number of members.

I'm thinking of a set whose members are an airplane and a horse.
Find a set to match mine.

SETS CONTAINING TWO MEMBERS

There are many things that we use almost every day that come in twos. Today, we will talk about some of those things. Look at the TWO shoes we are wearing.

Can you think of other things that we wear that come in twos?

The discussion of clothing that are worn in twos might include: socks, stockings, slippers, skates, boots, galoshes, shoe laces, mittens or gloves, etc.

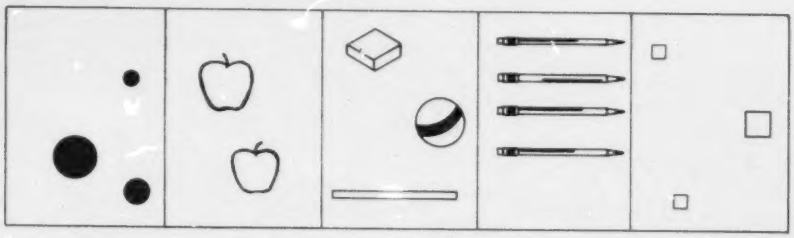
The singing game or chant "My Toes, My Knees, My Shoulders, My Head" is fun and points up the twoness of the other parts of the body since there is only one head.

•The children may enjoy having you read such poems as:

Roberts, Elizabeth Madox, "The Twins"
Milne, A. A., "Us Two"
Drinkwater, John, "Twos"

They may also enjoy hearing nursery rhymes that have two characters.

SETS EQUIVALENT TO A MODEL SET



A (1) (2) (3) (4)

Using materials for the magnetic board, perception cards, or felt pen on newsprint, arrange a set of three objects and at least three other sets so that at least one of these is equivalent to the first set. For example, sets of objects might appear as illustrated above where sets (2) and (4) are equivalent to set A. Select different children to find a set that matches the model set. Be sure that all of the sets that are equivalent to the model set are identified, along with the number of members in each.

Repeat this activity using sets having one, two, four, or five members.

Ask the children to find sets in the room that match the model set, whose number property is identified. Have each child pair the members of his set with the members of yours to see if one set has as many members as the other. Avoid having the children get the idea that the members of a set must be related in some way other than that they are members of the same set.

•Give each child a set of 5 objects.

Listen carefully as I describe the game that we will play today.

I will put a set of objects under this box (indicate) while your eyes are closed.

When I tell you to open your eyes, use the objects I gave you to form a set that you think is equivalent to the set under the box.

Place this set of objects in front of you.

Then I will remove the box and show you the set that was hidden.

Are you ready to play?

Close your eyes. I am placing a set under the box.
Open your eyes. Think.
Now, place in front of you a set that you think has as many members as the set under the box.

Look at my set.
Now look at your set.
If your set is equivalent to my set, please stand and tell me how many members there are in each set.

Repeat the game several times and ask various children to be helpers. Each helper should have a turn to select and hide a set for classmates to match.

- Place the following sets on a table or countertop: ten sets each containing four small objects; two sets of three objects; and one set of two objects. Show a set with four objects and ask for the number of members in this set.

Send two children to the table (countertop). As you slowly count to five, they must find a set with the number of members indicated and arrange it in a designated place for classmates to see. Continue until all of the sets of 4 objects have been used.

This activity may be repeated using sets with more or fewer members.

PROBLEM SOLVING

Give each child a set containing 5 small objects and a piece of 6" × 8" construction paper.

Today I'm going to tell you some stories.
You will use the set materials to answer the questions I'll ask.
After I ask the question, put your set answers under the paper so they won't be seen until it is time.

After you have given the problem and the children have decided upon answers, show the story using the magnetic board. Have the children check their answers and stand if their set is equivalent to the one on the board.

Mildred had two dolls.
Mother gave her one new doll.
How many dolls does Mildred have?

Jane has one book, one doll, and one top.
May I see how many toys Jane has?

Bill started to school with one ball.
He found another one on the way.
How many balls did Bill bring to school?

ADDITIONAL ACTIVITIES

1. Have the children scattered around the center of the room. At the sound of a bell or a chord of the piano, they are to form into sets with 1, 2, 3 or 4 members. Ask one child in each group to call out his set number.

2. There are some singing games and many songs that use numbers. "Ten Little Indians", "This Old Man", "Three Blind Mice", "Two Little Birds", "Six Little Chickadees", etc. Use these at music time.

3. Use perception cards illustrated in the back of the book for set descriptions of various types:
   (a) Recognizing sets with 1, 2, and 3 members.
   (b) Using sets of 2 and 3 members to name the number of a set with more members. The children may think of a set of 6 as 2 sets of 3 members each or as 3 sets of 2 members each.
   (c) Identifying the number of numbers in a subset within a set.
   (d) Pairing the members of two sets and discovering that one set has more than, fewer than, or as many members as the other. Mention number of members of each.
   (e) Joining sets and forming a new set. Mention number of members.
   (f) Removing a subset to find the remaining set. Mention number of members.
   (g) Recognizing that a set having three members arranged in a large space represents the same number as a set of three members within a small space.

   (h) Identifying two sets which have the same arrangement of members and also the same number of members.

MATERIALS

The following lists of materials suggest certain aids for use in the mathematics program. They are by no means exhaustive. You may find other materials that will make the work more meaningful to your particular class. It is not necessary that you have or use every item listed. Use your best judgment both in the selection of aids and in the extent of their use with your children.

MATERIALS FOR TEACHER DEMONSTRATION AND PUPIL USE

General classroom supplies: Paper clips, rubber bands, colored chalk, paper, tagboard, scissors, paste, crayons, paints, paint brushes, furniture, etc.

Balloons

Balls: a variety of sizes ranging from small ones to large beach balls

Blocks of various shapes and sizes

Books

Boxes of various sizes and shapes

Coordination board

Cookie or donut cutter

Embroidery hoop

Empty cans of various sizes

Fit-a-space

Flannel or magnetic board

Flannel or magnetic board objects

Form board

Games: dominoes, object lotto, puzzles, etc.

Geometric models made from a variety of materials such as: felt, construction paper, sandpaper, wire, styrofoam.

Graduated shapes

Hula hoop (if available)

Kittie in the kegs

Learning tower

Lincoln logs

Magnetic form board

Numeral cards

Old magazines

Paper bags

Parquetry blocks

Pegboard and pegs

Perception cards

Rhythm instruments

Seasonal materials: Indian corn; colored leaves; pumpkins; Christmas decorations; jelly eggs; greeting cards from various occasions; etc.

String, yarn or clothesline rope

Toys: dolls; cars; dishes; stand-up figures of family groups, community workers or storybook characters; dollclothes; trucks; etc.

Umbrellas of various sizes

Notice the next list, "Materials for Individual Pupils", and use as applicable.

MATERIALS FOR INDIVIDUAL PUPILS

MATERIALS

Beads and bead laces

Beans

Blocks (small colored)

Bottle caps

Buttons

Corks

Crayons

Cubes (one-inch-square wooden)

Disks

Figures (geometric, made from construction paper)

"Food for thought and stomach" (marshmallows, cookies, crackers, etc.)

General room supplies

Marbles

Paste sticks

Pegs

Plastic spoons and forks

Rocks and stones

Shells (macaroni or others)

Spools

Straws

String or yarn

Tongue depressors

Toys (small animals, cars, trucks, cowboys, surprises from cracker jacks and cereal boxes)

STORAGE BOXES

These may be made from any available materials. They are useful for storing each pupil's materials. The following ideas have been suggested.

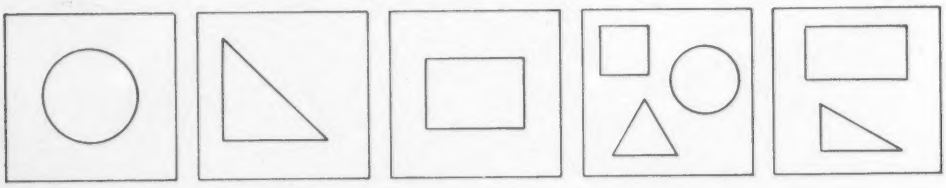
1. Boxes of various sizes which are convenient to store. If one wishes to, felt or flannel may be glued to the inside lid and thus create individual flannel boards for each pupil. Place the objects for use inside the box, and the children may manipulate these as directed or during free-time activities.
2. Cottage cheese cartons are easily obtained and very good for storage. They may be labeled for each child.
3. Paper bags or envelopes may be used as satisfactory containers. They can be used with an element of surprise: grab bag.

PERCEPTION CARDS

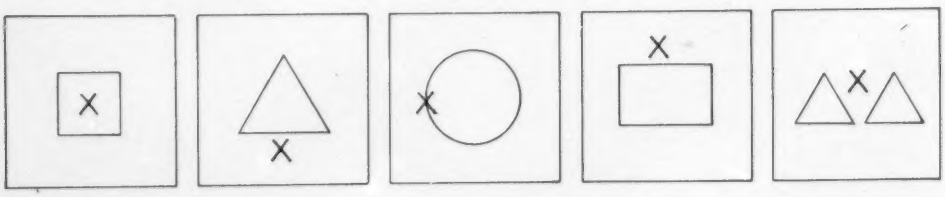
These aids showing sets of various members are excellent sources for reinforcing concepts and vocabulary. The cards may be made by the teacher for demonstration purposes and also by the children for class work. The latter individual cards are particularly valuable when working with "pairing, more than, larger than, same as etc." By keeping a variety of cards, the children will not tend to memorize the answers sought. At the same time interest and intrigue may result from the element of surprise. Some suggestions are listed below:

1. Pictures of sets from magazine advertisements, story and picture books. Mount these on heavy tagboard for demonstration purposes.

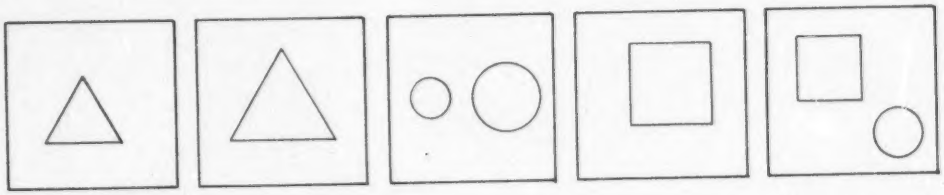
2. Geometric figures (individual and combinations) such as circle, triangles (three types), rectangle, and square.



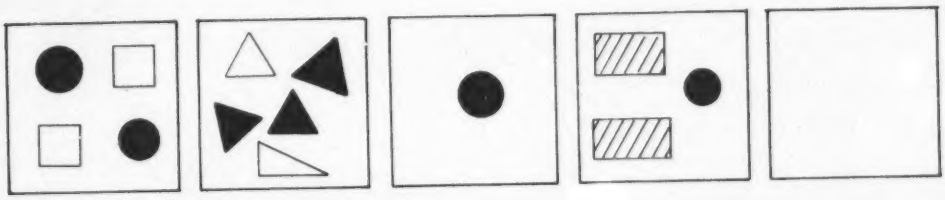
3. Various objects, such as geometric figures with an "X" placed in various positions to show "inside, outside, on, between, etc." For example: make a square and mark and "X" inside it. Ask the children where the "X" is located.



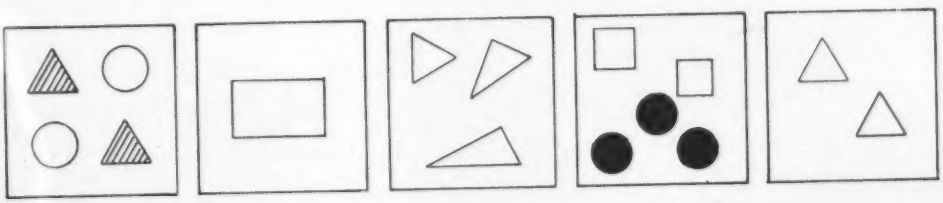
4. Smaller and larger figures to show comparison of size; individual or a complex group.



5. Sets of dots, geometric figures and other objects to compare set sizes (include a blank card depicting the empty set.)



6. Sets which can be easily partitioned into subsets of like members, one member sets, etc.



BIBLIOGRAPHY

Banner, Angela, One, Two, Three with Ant and Bee. New York: Franklin Watts, Inc., 1958.

Beim, Jerrold, Country Garage. New York: William Morrow and Co., 1952.

Berkley, Ethel S., Ups and Downs. Eau Claire, Wisconsin: E. M. Hale and Co., 1951.

_____, The Size of It. Eau Claire, Wisconsin: E. M. Hale and Co., 1951.

Bianco, Pamela, The Doll in the Window. New York: Henry Z. Walck, Inc., 1953.

Bishop, Claire Kutchet, The Five Chinese Brothers. New York: Coward-McCann, 1938.

Borten, Helen, Do You See What I See? New York: Abelard-Schuman, Ltd., 1950.

Budney, Blossom, A Kiss is Round. New York: Lothrop, 1954.

Brann, Esther, Five Puppies for Sale. New York: The MacMillan Co., 1948.

Corcos, Lucille, Joel Spends His Money. New York: Abelard-Schuman, Inc., 1954.

d'Aulaire, Ingri, The Two Cars. Garden City, New York: Doubleday and Co., Inc., 1955.

d'Aulaire, Ingri, and d'Aulaire, Edgar Parin, Don't Count Your Chicks. Garden City, New York: Doubleday and Co., Inc., 1943.

Duvoisin, Roger, Two Lonely Ducks, A Counting Book. New York: Alfred A. Knopf, Inc., 1955.

Emberley, Ed, The Wing on a Flea. Boston: Little, Brown, and Co., 1951.

Falconer, Rebecca, Tall-Enough Tommy. Chicago: Childrens Press, Inc., 1954.

Federico, Helen, The Golden Happy Books of Numbers. New York: Golden Press, 1963.

Friskey, Margaret, Chicken Little Count-to-Ten. Chicago: Childrens Press, Inc., 1946.

_____, Seven Diving Ducks. Philadelphia: David MacKay Co., 1940.

Geisel, Theodor Seuss (pseud. Dr. Seuss). The 500 Hats of Bartholomew Cubbins. New York: Vanguard Press, 1938.

_____, One Fish, Two Fish, Red Fish, Blue Fish. New York: Random House, 1960.

Hall, William, The Seven Little Elephants. New York: Thomas Y. Crowell Co., 1947.

Ipcar, Dahlow, Brown Cow Farm, A Counting Book. Garden City, New York: Doubleday and Co., 1959.

Jackson, Kathryn, Wheels. New York: Golden Books; Simon and Schuester, 1952.

Kay, Helen, One Mitten Lewis. New York: Lothrop, Lee, and Shepherd Co. Inc., 1955.

Kepes, Juliet, Two Little Birds. Boston: Houghton Mifflin Co., 1960.

Kipling, Rudyard, The Elephant's Child. Chicago: Rand McNally, 1955.

Kohn, Bernice, Everything Has a Shape. Englewood Cliffs, New Jersey: Prentice-Hall

Krasilovsky, Phyllis, The Very Little Girl. New York: Doubleday and Co. Inc., 1953.

Krauss, Ruth, The Growing Story. New York: Harper Brothers, 1947.

Kravetz, Nathan, Two for a Walk. New York: Henry Z. Walck, Inc., 1954.

Krum, Charlotte, The Four Riders. Chicago: Follet Publishing Co., 1953.

Mc Call, Adeline, Timothy's Tunes. Boston: Boston Music Company, 1948.

Meeks, Esther, One is the Engine. Chicago: Wilcox and Follett Co., 1952.

Moon, Grace and Carl, One Little Indian. Chicago: Albert Whitman and Co., 1952.

Moore, Lillian, Count to Ten. New York: Golden Press Inc., 1957.

Osswald, Edith, and Reed, Mary M., The Golden Picture Book of Numbers. New York: Simon and Schuster, 1954.

Paasche, Carol, Count With Me 1-2-3. New York: Golden Press, Inc., 1962.

Pease, Josephine Van Dolzen, One, Two, Cock-a-Doodle Doo. Chicago: Rand McNally Co., 1950.

Podendorf, Illa, The True Book of Pets. Chicago: Children's Press, 1954.

Rey, H. A., Feed the Animals. New York: Houghton Mifflin, 1944.

Roberts, Cliff, The Dot. New York: Franklin Watts Inc., 1960.

Schlein, Miriam, Fast is Not a Ladybug. New York: William R. Scott, Inc., 1953.

_____, The Four Little Foxes. New York: William R. Scott, Inc., 1953.

_____, Heavy is a Hippopotamus. New York: William R. Scott, Inc., 1954.

_____, Shapes. New York: William R. Scott, Inc., 1952.

Schneider, Herman and Nina, How Big is Big?. New York: William R. Scott, Inc., 1946.

Seignbobc, Francoise, Jeanne Marie Counts her Sheep. New York: Charles Scribner's Sons, 1951.

Skarr, Grace, All About Dogs, Dogs, Dogs. Eau Claire, Wisconsin: E. M. Hale and Co., 1954.

Slobodkin, Louis, One is Good but Two Are Better. New York: Vanguard Press, Inc., 1956.

Smock, Nell S., Little and Big. Nashville, Tennessee: Abingdon-Cokesbury Press, 1947.

True, Louise, Number Men. Chicago: Childrens Press Inc., 1948.

Tudor, Tasha, 1 is One. New York: Henry Z. Walck, Inc., 1956.

Ungirer, Tomi, One, Two, Where's My Shoe. New York: Harper and Row, Pub., 1964.

Watson, Nancy, What is One?. New York: Alfred A. Knopf, 1954.

Wolff, Janet and Owett, Bernard, Let's Imagine Thinking Up Things. New York: E. P. Dutton and Co., Inc., 1961.

Zinzer, Feenie and Galdone, Paul. Counting Carnival. New York: Coward-McCann, Inc., 1962.

KINDERGARTEN VOCABULARY

Your children will be introduced to many new words during the course of the year. Many of these words will be well understood and used by the children. However, other words will need constant reinforcement as the child progresses through the primary grades. We hope that your children will have some understanding of the words below by the end of the kindergarten.

above
alike
around
as many as
behind
below
beside
between
big
bigger than
biggest
bottom
center
circle
close
closer
closest
collection
corner
different
edge
empty set
equivalent
fewer than
fewest
first
five
four
high
higher than
highest
in
inside
join
large
larger than
largest
last
left
long
longer than
longest
match
member
middle
more than
most
narrow
number
on
one
order
outside
over
pair
pairing
position
rectangle
region
remaining set
remove
right
same
set
shape
short
shorter than
shortest
side
small
smaller than
smallest
square
subset
tall
taller than
tallest
thick
three
top
triangle
two
under
wide